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Solutions to

1.1 Problems in Quantum Mechanics A1-A16

Last corrections 29/11 2010

A1 $\sqrt{\frac{\pi}{a}}\ e^{ik_0 x}\ e^{-\frac{x^2}{4a}}$

A2 QM: $F = \frac{2E}{L}$, CF: gives the same.

A3 $|R|^2 = \left(\frac{\sqrt{E+V_0}-\sqrt{E}}{\sqrt{E+V_0}+\sqrt{E}}\right)^2$, $|T|^2 = \frac{4\sqrt{E(E+V_0)}}{(\sqrt{E+V_0}+\sqrt{E})^2}$

A4 $R = \frac{(k^2+\kappa^2)(e^{\kappa a}-e^{-\kappa a})}{2ik\kappa(e^{\kappa a}-e^{-\kappa a})+(k^2-\kappa^2)(e^{\kappa a}-e^{-\kappa a})}$

A5 -.

A6 $|T|^2 \approx exp(-2\pi\sqrt{\frac{m}{\kappa}}(-E)/\hbar)$

A7 proof.

A8 proof.

A9 $u(x,y,z) = \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-(x^2+y^2+z^2)/2b^2}$, where $b = \sqrt{\frac{\hbar}{m\omega}}$ and $u(r,\theta,\varphi) = \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-r^2/2b^2}$

A10 $\frac{\hbar}{i}\cos\varphi$

A11 Use $\mathrm{Y}_{l,-m} = (-1)^m\ \mathrm{Y}_{l,m}$

A12 a) $Pf(r,\theta,\varphi) = f(r,\pi-\theta,\varphi+\pi)$ b) $P\mathrm{Y}_{l,l} = (-1)^l\ \mathrm{Y}_{l,l}$ c) proof

A13 a) $\langle u_{100}|z|u_{200}\rangle = 0$ b) $\langle u_{100}|z|u_{210}\rangle = \frac{1}{\sqrt{2}}\left(\frac{2}{3}\right)^6 24\frac{a_0}{Z}$

A14 Eigenstates are

$$\chi_{\varphi+} = \begin{pmatrix} \frac{e^{-i\varphi/2}}{\sqrt{2}} \\ \\ \frac{e^{i\varphi/2}}{\sqrt{2}} \end{pmatrix} \tag{1.1}$$

and

$$\chi_{\varphi-} = \begin{pmatrix} \frac{e^{-i\varphi/2}}{\sqrt{2}} \\ \\ \frac{-e^{i\varphi/2}}{\sqrt{2}} \end{pmatrix} \tag{1.2}$$

A15 $\frac{1}{2}$

A16 Eigenenergys are $E_+ = -3.53 \cdot 10^{-26}$J and $E_- = +3.53 \cdot 10^{-26}$J.

$$\chi(t) = A \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{+i\ 3.34 \cdot 10^{+8}\ t} \ + \ B \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-i\ 3.34 \cdot 10^{+8}\ t} \tag{1.3}$$

and subject to $|A|^2 + |B|^2 = 1$.

A17 17.6 MeV

A18 proof

A19 result for $\Psi(x) = N\frac{\pi}{\alpha}e^{-\alpha|x|}$, use it to make your estimates.

A20 $\phi(p) = \frac{A}{\sqrt{2\pi}}\frac{2\mu}{\mu^2+k^2}$

A21 $A = \sqrt{\mu}$

A22 proof

A23 $\Delta E = 1.14$ eV, $1.085 \cdot 10^{-6}$m.

A24 n $= 4 \cdot 10^7$, $7.6 \cdot 10^{-8}$eV

A25 No, $P_1 = \frac{4}{\pi^2}$ and $P_2 = \frac{4}{\pi^2}$.

A26 a) $\Psi(x,t) = A\sqrt{\frac{a}{2}}\frac{1}{16}\left(u_5(x)e^{-iE_5t/\hbar} - 5u_3(x)e^{-iE_3t/\hbar} + 10u_1(x)e^{-iE_1t/\hbar}\right)$ b) $A^2 = \frac{256}{63a}$, c) 25/126 .

A27 Width $\approx 1/\sqrt{\alpha}$, gives $E \approx (\Delta p)^2/2m = \frac{\alpha\hbar^2}{2m}$