LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Solution to written exam in SOLID STATE PHYSICS F0019T AND F7006T

Examination date: 2015-08-27
The solutions are just suggestions. They may contain several alternative routes.
This is a combined solution for the courses F0019T and F7006T.
For F0019T use solution 1, 2, 3, 4 and 5.
For F7006T use solution 2, 3, 4, 5 and 6 with an offset of -1.

1. (a) There are several ways to determine A. One is to integrate and use the normalization condition to solve for A. A different path (done here) is to write the given wave function in terms of eigenfunctions (here particle in a box). The eigenfunctions are (PH) $\psi_n(x) = \sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a})$. We can directly conclude that the given wave function consists of $n = 1$, $n = 5$ and $n = 7$ functions, we can write:

$$\psi(x,0) = \frac{\sqrt{13}}{\sqrt{8\cdot 2}}\frac{\sqrt{2}}{\sqrt{a}}\sin\left(\frac{\pi x}{a}\right) + \frac{\sqrt{2}}{2\sqrt{2\cdot a}}\sin\left(\frac{5\pi x}{a}\right) + \frac{A\sqrt{2}}{\sqrt{2\cdot a}}\sin\left(\frac{7\pi x}{a}\right) =$$

$$\frac{\sqrt{13}}{\sqrt{16}}\psi_1(x,0) + \frac{1}{\sqrt{8}}\psi_5(x,0) + \frac{A}{\sqrt{2}}\psi_7(x,0)$$

As all three eigenfunctions are orthonormal the normalisation integral reduces to $\frac{13}{16} + \frac{1}{8} + \frac{A^2}{2} = 1$ and hence $A = \sqrt{\frac{1}{8}}$ (≈ 0.354).

(b) The wave function contains only $n = 1$, $n = 5$ and $n = 7$ eigenfunctions and therefore the only possible outcome of an energy meassurement are $E_1 = \frac{\hbar^2\pi^2}{2ma^2}$ with probability $\frac{A^2}{4} = \frac{13}{16}$ and $E_5 = \frac{\hbar^2\pi^2}{2ma^2}25$ with probability $\frac{1}{8}$ and $E_7 = \frac{\hbar^2\pi^2}{2ma^2}49$ with probability $\frac{1}{16}$.
The average energy is given by
$< E >= \frac{13}{16}E_1 + \frac{1}{8}E_5 + \frac{1}{16}E_7 = \frac{\hbar^2\pi^2}{2ma^2}(\frac{13}{16} + \frac{1}{8}\cdot 25 + \frac{1}{16}\cdot 49) = \frac{112}{16}\cdot\frac{\hbar^2\pi^2}{2ma^2} = 7\cdot\frac{\hbar^2\pi^2}{2ma^2}$

(c) The time dependent solution is given by $\Psi(x,t) = \sum_{n=1}^{\infty} c_n\psi_n(x)e^{-iE_nt/\hbar}$ and hence

$$\Psi(x,t) = \sqrt{\frac{13}{16}}\psi_1(x,0)e^{-i\frac{\hbar\pi^2 t}{2ma^2}} + \frac{1}{\sqrt{8}}\psi_5(x,0)e^{-i\frac{25\hbar\pi^2 t}{2ma^2}} + \frac{1}{4}\psi_7(x,0)e^{-i\frac{49\hbar\pi^2 t}{2ma^2}}$$

2. $\Theta_D = 148$K . Lutning i C_v/T vs T^2 figur är 0.609 .

3. $m_h = +5\ 10^{-32}$kg, $\epsilon_h = +10^{-19}$J, $\mathbf{p}_h = -10^{-25}\hat{\mathbf{k}}_x$kg m/s, $\mathbf{v}_h = -2\ 10^6\hat{\mathbf{k}}_x$m/s

4. **rectangular unit cell, the q are lattice points**, *basis of letters associated with each lattice point*, *PRIMITIVE UNIT CELL.*

q p d b q p d b q p d b *Q* p d b . . .

d b q p d b *q p* d b *Q* p d b *Q* p . . .

q p d b q p *d b* q p d b *Q* p d b . . .

d b q p d b q p d b q p d b q p

. . .
. . .
. . .

5. Den intrinsiska ledningsförmågan ges av $AT^{3/2}e^{-E_g/2k_BT} = 7 \cdot 10^{21}T^{3/2}e^{-6400/T}$; konstanten A väljs med data för T=300 K. Provet upphör att visa egenledning då n_i är av samma storleksordning som n_d, dvs $T \leq 360$ K.

6. En analys av strukturfaktorn S_{hkl} för diamant ger följande (fcc med bas 000 och $\frac{1}{4}\frac{1}{4}\frac{1}{4}$):
$S_{hkl} = \sum_i f_i e^{-2\pi i(hx_i+ky_i+lz_i)} = f_C \left(1 + e^{-\frac{\pi}{2}i(h+k+l)}\right)\left(1 + e^{-\pi i(h+k)} + e^{-\pi i(k+l)} + e^{-\pi i(h+l)}\right)$ och $S_{hkl} \neq 0$ om $h+k+l = 4 \cdot$ heltal och $S_{hkl} \neq 0$ om alla h, k, l är udda eller jämna. Följande serie för hkl erhålles. 111, 220, 311, 400, 331... För Braggspridning gäller $2d_{hkl}\sin\theta = \lambda$ där $d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$ dvs $d = \frac{\lambda}{2\sin\theta}$ och $\frac{1}{d^2} \sim h^2 + k^2 + l^2$ vilket ger följande analys

θ	21.4	36.6	44.5	57.5
d	2.0555	1.25791	1.0700	0.88927
$\frac{1}{d^2}$	0.23668	0.63197	0.87378	1.2645
$x = \frac{3}{0.23668}$	3	8.010	11.07	16.028
$h^2 + k^2 + l^2$	3	8	11	16
(hkl)	111	220	311	400

Detta ger att ämnet har diamantstruktur och enhetskubenskantlängd a blir $d = 2.0555 = \frac{a}{\sqrt{3}}$ vilket ger $a = 3.56$Å.