LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Solution to written exam in SOLID STATE PHYSICS F0053T AND F7045T

Examination date: 2016-05-31
The solutions are just suggestions. They may contain several alternative routes.
This is a combined solution for the courses F0053T and F7045T.
For F0053T use solutions 1, 2, 4, 5 and 3.
For F7045T use solutions 4, 5, 6, 7 and 3.

1. As we are dealing with a carbon rich area we as a first guess go for carbon with just one electron left (= hydrogen like). The Carbon ion has $Z = 6$ and hence energys $E_n = -\frac{488.16}{n^2}$eV. Try to find a start of the series. The energy of $\lambda = 207.80$nm is
$E = h\nu = \frac{hc}{\lambda} = \frac{6.626 \cdot 10^{-34} \cdot 2.9979 \cdot 10^8}{207.80 \cdot 10^{-9} \cdot 1.6022 \cdot 10^{-19}} = 5.9663$eV A similar calculation gives the energys for the other lines in the series: 9.56395, 11.8989 and 13.4997 eV.

   As the Balmer series in Hydrogen is for transitions down to level n=2 we have to go higher up for the Carbon ion as the energys for the level $n = 2$ in Carbon would be far to large.

   Using the fact that can assume levels are adjecent we let n be the quantum number for the lower level and m for a level above, we have no knowledge of how n and m relate. We know however that for the next level (higher in energy) we have n and $m + 1$. One can form the following two equations 5.9663eV=488.16($\frac{1}{n^2} - \frac{1}{m^2}$)eV and 9.56395eV=488.16($\frac{1}{n^2} - \frac{1}{(m+1)^2}$)eV ie we only need two of the lines to form an appropriate set of equations. (You can use the other pairs of lines as well to form two equations.) Subtracting one equation from the other to eliminat n you get 3.59765=488.16($\frac{1}{m^2} - \frac{1}{(m+1)^2}$) and $\frac{1}{m^2} - \frac{1}{(m+1)^2} = 0.007369817273025237627$ solving for m you arrive at $m = 6$. Now we use the result for m in 5.9663eV=488.16($\frac{1}{n^2} - \frac{1}{6^2}$)eV to solve for n and we arrive at $n = 5$.

   Then there is the tour of brute force ie just trial and error: If we try n=5 we have transitions from m=6, 7, 8, 9, etc. The corresponding energys will be: 488.16($\frac{1}{5^2} - \frac{1}{6^2}$)=5.97 eV, the next one will be: 488.16($\frac{1}{5^2} - \frac{1}{7^2}$)=9.56 eV, 488.16($\frac{1}{5^2} - \frac{1}{8^2}$)=11.899 eV and so on. So these are down to n=5 from level m=6, 7, 8 and 9.

2. (a) There are several ways to determine A. One is to integrate and use the normalization condition to solve for A. A different path (done here) is to write the given wave function in terms of eigenfunctions. The eigenfunctions are (PH) $\psi(x) = \sqrt{\frac{2}{a}} \sin(\frac{n\pi x}{a})$. We can directly conclude that the given wave function consists of eigenfunctions with $n = 1$ and $n = 5$, we can write:

   $$\psi(x,0) = \frac{A\sqrt{2}}{\sqrt{2a}} \sin\left(\frac{\pi x}{a}\right) + \frac{\sqrt{2}}{\sqrt{2 \cdot 5a}} \sin\left(\frac{5\pi x}{a}\right) = \frac{A}{\sqrt{2}}\psi_1(x,0) + \frac{1}{\sqrt{10}}\psi_5(x,0)$$

   As both eigenfunctions are orthonormal the normalisation integral reduces to $\frac{A^2}{2} + \frac{1}{10} = 1$ and hence $A = \sqrt{\frac{18}{10}} = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$

   (b) The wave function contains only $n = 1$ and $n = 5$ eigenfunctions and therefore the only possible outcomes of an energy meassurement are $E_1 = \frac{\hbar^2\pi^2}{2ma^2}$ with probability $\frac{A^2}{2} = 0.9$ and $E_5 = \frac{\hbar^2\pi^2}{2ma^2}25$ with probability 1 - 0.9 = 0.1 . The average energy is given by
   $< E >= 0.9E_1 + 0.1E_5 = \frac{\hbar^2\pi^2}{2ma^2}(0.9 + 0.1 \cdot 25) = 3.4 \cdot \frac{\hbar^2\pi^2}{2ma^2} = 1.7 \cdot \frac{\hbar^2\pi^2}{ma^2}$

(c) The time dependent solution is given by $\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_n t/\hbar}$ and hence

$$\Psi(x,t) = \sqrt{\frac{9}{10}} \psi_1(x,0) e^{-i\frac{\hbar\pi^2 t}{2ma^2}} + \frac{1}{\sqrt{10}} \psi_5(x,0) e^{-i\frac{25\hbar\pi^2 t}{2ma^2}}$$

3. The Angle $\beta = 2\theta$.

β	θ	$\sin^2\theta_i / \sin^2\theta_1$	X_i	(hkl)
30.3	15.15	1.00	2	(110)
43.4	21.7	2.00	4	(200)
53.9	26.95	3.00	6	(211)
63.1	31.55	4.00	8	(220)
71.6	35.8	5.00	10	(310)
79.7	39.85	6.00	12	(222)
87.6	43.8	7.00	14	(321)

It is not possible to find integers that gives $X_i = h^2 + k^2 + l^2 = 7$. Therefore the values are multiplied by two. We see that all planes have $h + k + l = 2n$ (even integer), which means that the structure is **bcc**.

4. $C_v = C_v^{el} + C_v^{ph}$. As the temperature in question (300K) is well above the Debye temperature (160K) we can use Dulong-Petits law for the phonons $C_v^{ph} = 3Nk_B$. For the electron contribution $C_v^{el} = \frac{\pi^2}{2} Nk_B \frac{T}{T_F}$, $T_F = E_F/k_B$ and $E_F = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V}\right)^{2/3}$. For Na we have $\rho = 971 kg/m^3$, atomic weight $= 22.9898$u some calculations gives $T_F = 36599.353$K. Fraction contributed by the electrons: $F = \frac{C_v^{el}}{C_v^{el} + C_v^{ph}} = \frac{1}{1 + \frac{6T_F}{\pi^2 T}} \approx 0.0133$.

5. (a) Primitiva enhetscellen: Al fcc: 1, Cr bcc: 1, Germanium diamantstruktur: 2.

   (b) Kubiska enhetscellen: Al fcc: 4, Cr bcc: 2, Germanium diamantstruktur: 8.

   (c) Germanium diamant struktur med kantlängden $a = 5.658$Å på enhetskuben. Närmsta grannar i tex. (0,0,0) och $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ detta avstånd $= \sqrt{3}a/4 = 2.45$Å. Näst närmsta grannar i tex. (0,0,0) och $(\frac{1}{2}, \frac{1}{2}, 0)$ detta avstånd $= \sqrt{2}a/2 = 4.00$Å.

6. (a) $E_d = -\frac{m_e e^4}{2\hbar^2 (4\pi\epsilon_r \epsilon_0)^2} = 6.6 \ 10^{-4}$eV.

   (b) $r = \frac{\epsilon_r \hbar^2}{m_e e^2} 4\pi\epsilon_0 = 642$ Å.

   (c) Överlappet blir betydande om koncentrationen $N_d \sim \frac{1}{(2r)^3} \approx 10^{21}$m^{-3}.

   (d) I likhet med hur atomära nivåer bildar band då atomer sammanförs till en kristall kommer donator nivåerna att bilda band då koncentrationen blir så stor att de lokala banorna överlappar.

7. The conductivity is given by $\sigma = ne\mu_e + pe\mu_h$. Asuming we are in the intrinsic regime we have for the denisty of electrons $n = p = n_i$ and hence the resistance is given by

$$R \propto 1/\sigma \propto n_i \propto e^{E_g/2k_BT}.$$

We only keep track of the exponentials and assume the algebraic expresion in T in n_i does not change alot compared to the exponential. If the sample is intrinsic will show in a graph of $\ln(R)$ vs $1/T$.

To analyse the data we note that

$$\ln(R) \propto \frac{E_g}{2k_B} \frac{1}{T}.$$

In the table below data is processed accordingly:

$T(^oC)$	R	T (K)	$1/T$	$\ln(R)$
22.0	182	295	$3.390 \cdot 10^{-3}$	5.204
48.0	92.0	321	$3.115 \cdot 10^{-3}$	4.522
72.0	53.0	345	$2.899 \cdot 10^{-3}$	3.970
97.0	32.0	370	$2.703 \cdot 10^{-3}$	3.466
127	17.2	400	$2.500 \cdot 10^{-3}$	2.845

If we draw $\ln(R)$ as a function of $1/T$ and if the data follows a straight line we calculate its slope.

$$\frac{E_g}{2k_B} = \frac{5.204 - 2.845}{(3.390 - 2.500) \cdot 10^{-3}} = 2650.6$$

and hence we can calculate E_g accordingly:

$$E_g = \frac{2650.6 \cdot 2 \cdot 1.38 \cdot 10^{-23}}{1.602 \cdot 10^{-19}} = 0.457\text{eV}.$$

$E_g = 0.46\text{eV}$