LULEA UNIVERSITY OF TECHNOLOGY
Applied Physics

| Course code | F0019T |
| :--- | :--- |
| Examination date | $2014-06-05$ |
| Time | $9.00-14.00$ (5 hours) |

Examination in: FASTA TILLSTÅNDETS FYSIK MED KVANTMEKANIK / QUANTUM Mechanics and Solid State Physics
Total number of problems: 5
Teacher on duty: Hans Weber
Tel: (49)2088, Room E304
Examiner: Hans Weber
Tel: (49)2088, Room E304
Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, Collection of formulae for Solid state physics and Collection of formulae for Quantum Physics.

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p .7 .5 points are required to pass the examination. Grades 3: 7.5, 4: 9.5, 5: 12.0

## 1. Operators and eigenfunctions

Are the following functions $\psi$ eigenfunctions of the given operators $\hat{A}$ ?
Note: Do not make guesses, a wrong answer will be counted negative! (No answer to one of the items is counted as zero)
(a) $\psi(t)=\cos \omega t$ and $\hat{A}=i \hbar \frac{\partial^{2}}{\partial t^{2}}$.
(b) $\psi(x)=e^{i k x}$ and $\hat{A}=\frac{\partial}{\partial x}$.
(c) $\psi(x)=e^{-a x^{2}}$ and $\hat{A}=\frac{\partial}{\partial x}$.
(d) $\psi(x)=\cos k x$ and $\hat{A}=\frac{\partial}{\partial x}$.
(e) $\psi(x)=k x$ and $\hat{A}=\frac{\partial}{\partial x}$.
(f) $\psi(x)=\sin k x$ and $\hat{A}=\hat{P}=$ the parity operator.
(g) $\psi(z)=C\left(1+z^{2}\right)$ and $\hat{A}=-i \hbar \frac{\partial}{\partial z}$.
(h) $\psi(z)=C e^{-3 z}$ and $\hat{A}=-i \frac{\hbar}{2} \frac{\partial}{\partial z}$.
(i) $\psi(z)=C z e^{-\frac{1}{2} z^{2}}$ and $\hat{A}=\frac{1}{2}\left(z^{2}-\frac{\partial^{2}}{\partial z^{2}}\right)$.
2. Crystal structure and the reciprocal lattice

Three two-dimensional structures (A, B and C) are shown in Figure 1 (you may assume that in each case the pattern is repeated to in- finity).
(a) For each structure write down a set of primitive lattice vectors, and briefly describe the primitive basis.


Figure 1: Three structures A, B and C. The markings • and o represent different kinds of atoms.
(b) Write down the reciprocal lattice vectors for structure A.
(c) For structure A show that Bragg reflection can occur when $k-k^{\prime}=n(2 \pi / a) \hat{i}+m(2 \pi / b) \hat{j}$, where $n$ and $m$ are integers. The wavevectors of the incoming and outgoing beams are $k$ and $k^{\prime}$.

## 3. Debye temperature

The table below shows measurements of the heat capacity of Zn . Use the data to calculate the Debye temperature.

| $T(\mathrm{~K})$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $C_{v}(\mathrm{~J} / \mathrm{kmol} \mathrm{K})$ | 0.72 | 1.83 | 3.80 | 7.19 | 12.0 |



Figure 2: Principal drawing of the scattering settup

## 4. X-ray diffraction

Below you find data from a measurement of the x-ray diffraction pattern from a powder sample. The table shows the angles $\beta$ where the diffraction peaks are found. Identify the cubic crystal structure. In Figure 2 the setup is shown.

| $\beta$ | $30.3^{\circ}$ | $43.4^{\circ}$ | $53.9^{\circ}$ | $63.1^{\circ}$ | $71.6^{\circ}$ | $79.7^{\circ}$ | $87.6^{\circ}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## 5. Semiconductor

The electrical conductivity of semiconductors can be changed if impurities are introduced in the crystal. Find out how the electrical conductivity of silicon $(\mathrm{Si})$ is changed if we introduce a small amount of antimony ( Sb ) in the crystal. The silicon crystal is doped in such a way that there is one Sb atom per $10^{6} \mathrm{Si}$ atoms. Is it true that the conductivity at 300 K of the doped silicon is 1000 times higher than the conductivity of a pure silicon crystal?

## Good Luck!

