LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0053T / F0019T (old)**
Examination date	2016-08-27
Time	9.00 - 14.00 (5 hours)

Examination in: FASTA TILLSTÅNDETS FYSIK MED KVANTMEKANIK / QUANTUM MECHANICS AND SOLID STATE PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE for Solid state physics and COLLECTION OF FORMULAE for Quantum Physics.

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 9.5, 5: 12.0

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1. **Heat capacity**

A measurement of the heat capcity of Potasium (Kalium) at low tempertures the following results where reached: (Potasium has one valence electron, $E_F = k_B T_F$.)

T (K)	0.5	1.0	1.5	2.0	2.5	3.0	3.5
C_v (mJ/mol K)	1.38	5.00	13.1	28.0	52.09	87.0	136

(a) Using these data determine the Debye temperture θ_D and the Fermi energy E_F of potassium.

(b) Using crystal data and the experimental results for (E_F) determine the effective mass of the electrons in terms of the free electron mass m_0.

(3p)

2. **Reciprocal space.**

(a) For sodium (natrium) calculate the shortest distance in reciprocal space from the origin to the surface of the Brillouin zone.

(b) Is the Fermi sphere larger or smaller than the Brillouin zone and by how much?

(3p)

TURN PAGE!

3. **Bragg scattering**

Silicon (Si) and Galliumarsenid (GaAs) both have the same primitive lattice structure, fcc, with the following basises:

Si	(000),	$(\frac{1}{4}\frac{1}{4}\frac{1}{4})$
GaAs	Ga (000),	As $(\frac{1}{4}\frac{1}{4}\frac{1}{4})$

Determine the the Miller indexies for the first four allowed Bragg reflections with the smallest difraction angles (glansvinkel). The atomic formfactors are not equal for any of the different atoms.

(3p)

4. **Two dimensional Square well**

A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 \leq x \leq a \text{ and } 0 \leq y \leq \frac{a}{2} \\ +\infty & \text{for} \quad x > a,\ x < 0 \text{ and } y > \frac{a}{2},\ y < 0. \end{cases}$$

(a) Calculate (solve the Schrödinger equation) the eigenfunctions $\psi_{n,m}(x,y)$!

(b) Write down the eigenfunctions for the ground state $\psi_{1,1}(x,y)$ and one for the lowest excited states ($\psi_{2,1}(x,y)$ or $\psi_{1,2}(x,y)$). Formulate the meaning of orthogonallity and show by explicit calculation that these two eigenfunctions are orthogonal.

(3p)

5. **Hydrogen atom**

Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

$$\Psi(\mathbf{r}, t=0) = \frac{1}{\sqrt{15}} \left(3\psi_{100}(\mathbf{r}) - 2\psi_{200}(\mathbf{r}) + \psi_{320}(\mathbf{r}) - \psi_{322}(\mathbf{r})\right)$$

(a) Is this wave function an eigenfunction of the parity operator $\hat{\Pi}$?

(b) What is the probability of finding the system in the ground state? In the state (200)? In the state (320)? In the state (322)? In any other state?

(c) What is the expectation value of the energy (in eV); of the operator $\mathbf{L}^2$ (in units of $\hbar^2$); of the the operator L_z (in units of $\hbar$).

(3p)

Good Luck !