LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0053T / F0019T (old)**
Examination date	2017-06-02
Time	9.00 - 14.00 (5 hours)

Examination in: FASTA TILLSTÅNDETS FYSIK MED KVANTMEKANIK / QUANTUM MECHANICS AND SOLID STATE PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber — Tel: (49)2088, Room E163
Examiner: Hans Weber — Tel: (49)2088, Room E163

Allowed aids: Fysika(lia), Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE for Solid state physics and COLLECTION OF FORMULAE for Quantum Physics.

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 9.5, 5: 12.0

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1. **Crystal structure and the reciprocal lattice**

   Three two-dimensional structures (A, B and C) are shown in Figure 1 (you may assume that in each case the pattern is repeated to in- finity).

   (a) For each structure write down a set of primitive lattice vectors, and briefly describe the primitive basis.

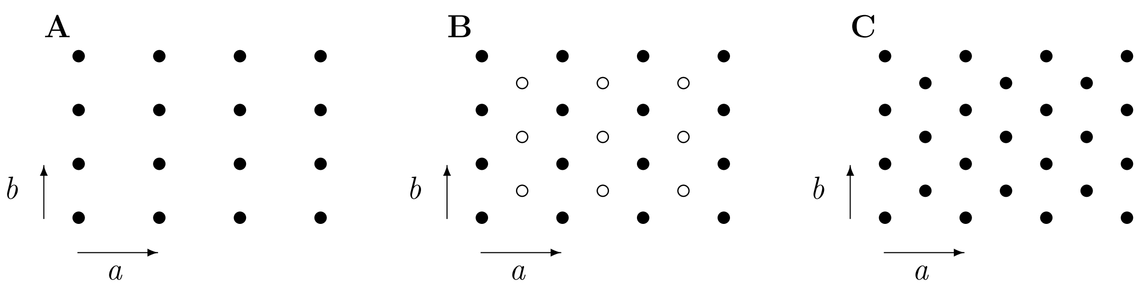

Figure 1: Three structures A, B and C. The markings • and ○ represent different kinds of atoms.

   (b) Write down the reciprocal lattice vectors for structure A.

   (c) For structure A show that Bragg reflection can occur when $k - k' = n(2\pi/a)\hat{\imath} + m(2\pi/b)\hat{\jmath}$, where n and m are integers. The wavevectors of the incoming and outgoing beams are k and k'. (3p)

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2. **Heat capacity**

The figure 2 shows the temperature dependence of the heat capacity C_v for Potassium (K).

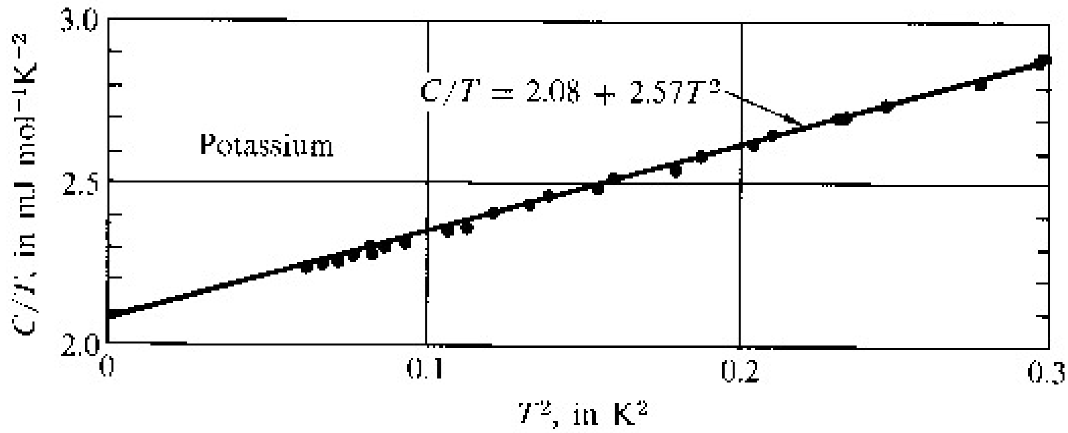

Figure 2: The specific heat of Potassium at low temperatures. The figure shows C_v/T (in mJ/(mole K^2) as a function of T^2.

(a) What are the theories that explain a plot of C_v/T as a function of T^2 is linear.

(b) Use the data from the figure and crystal data to determine the Debye temperature θ_D.

(c) Use the data from the figure and crystal data to determine the efective mass of the electrons expressed in terms of the free electron mass m_0. The Fermi energy is given by $E_F = k_B T_F$.

(3p)

3. **Bragg analysis**

The powders of three different substances, all mono atmic cubic crystals, are analysed with a Debye–Scherrer camera. We also know that one of the powders has fcc, one ha bcc and one has diamond structure. The angular positions (ϕ in degrees) of the first four diffraction rings are given in the table below. In figure 3 a principle drawing of a Debye-Scherrer camera is shown.

A	B	C
42.2	28.8	42.8
49.2	41.0	73.2
72.2	50.8	89.0
87.3	59.6	115.0

(a) Identify the crystal structures for samples A, B and C.

(b) If the wavelength of X–rays is $\lambda = 1.50$Å, what would the lattice constant be for the 3 different samples?

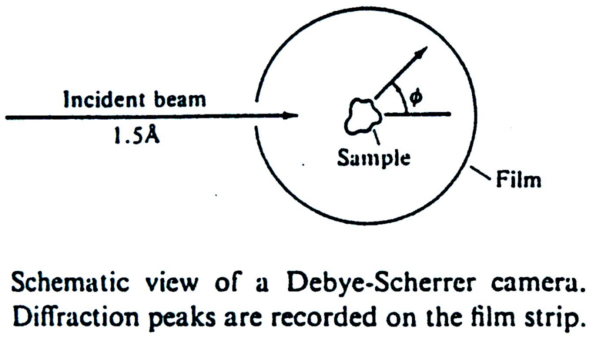

Figure 3: A principle figure of the geometry of a Debye-Scherrer camera.

(3p)

4. **Operators and eigenfunctions**

Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

Note: Do not make guesses, a wrong answer will be counted negative ! (No answer to one of the items is counted as zero)

(a) $\psi(t) = \cos\omega t$ and $\hat{A} = i\hbar\frac{\partial^2}{\partial t^2}$.
(b) $\psi(x) = e^{ikx}$ and $\hat{A} = \frac{\partial}{\partial x}$.
(c) $\psi(x) = e^{-ax^2}$ and $\hat{A} = \frac{\partial}{\partial x}$.
(d) $\psi(x) = \cos kx$ and $\hat{A} = \frac{\partial}{\partial x}$.
(e) $\psi(x) = kx$ and $\hat{A} = \frac{\partial}{\partial x}$.
(f) $\psi(x) = \sin kx$ and $\hat{A} = \hat{P} =$ the parity operator.
(g) $\psi(z) = C(1+z^2)$ and $\hat{A} = -i\hbar\frac{\partial}{\partial z}$.
(h) $\psi(z) = Ce^{-3z}$ and $\hat{A} = -i\frac{\hbar}{2}\frac{\partial}{\partial z}$.
(i) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

(0 – 3 p)

5. **Hydrogen atom**

Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

$$\Psi(\mathbf{r}, t=0) = A\left(2\psi_{100}(\mathbf{r}) - 3\psi_{211}(\mathbf{r}) + \psi_{320}(\mathbf{r}) - \psi_{322}(\mathbf{r})\right)$$

(a) Determine A so that the wave function is normalised.
(b) What is the probability of finding the system in the ground state? In the state (211)? In the state (320)? In the state (322)? In any other state?
(c) What is the expectation value of the energy (in eV); of the operator $\mathbf{L}^2$ (in units of $\hbar^2$); of the the operator L_z (in units of $\hbar$).

(3p)

Good Luck !