LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0053T / F0019T (old)**
Examination date	2017-08-26
Time	9.00 - 14.00 (5 hours)

Examination in: Fasta tillståndets fysik med kvantmekanik / Quantum Mechanics and Solid State Physics
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysika(lia), Physics Handbook, Beta, calculator, Collection of formulae for Solid state physics and Collection of formulae for Quantum Physics.

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 9.5, 5: 12.0

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1. **Reciprocal space.**

   (a) For rubidium calculate the shortest distance in reciprocal space from the origin to the surface of the Brillouin zone.

   (b) Is the Fermi sphere larger or smaller than the Brillouin zone and by how much?

   (3p)

2. **The specific heat of Gold**

   A measurement of the heat capacity C_v is performed. The results are given in the table below:

T (K)	1.6	2.0	2.4	2.8	3.2	3.6
C_v (J /kmol K)	4.18	6.88	10.7	15.9	23.0	31.8

   Use these experimental results to determine the debye temperature Θ_D for Gold. (3p)

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3. **A primitive cell consisting of two atoms**

The structure of NaCl is FCC with the basis one Na at (0,0,0) and one Cl at $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. The lattice constant of the cubic unit cell is $a = 5.65$ Å. The velocity of sound for a transverse acoustic wave in the [111] direction is $2.75 \cdot 10^3$ m/s. As each (111) plane in NaCl only consists of atoms of the same kind one can treat lattice vibrations with a simple one dimensional model where each plane is represented by one atom.

(a) Make a drawing of the one dimensional crystal and determine the size of the primitive unit cell in terms of the lattice constant a, and sketch the the dispersion relation (in the [111] direction) on a principal level.

(b) Use the group velocity $v = \partial\omega/\partial K$, and the expansion of the dispersion relation

$$\omega^2 = C\frac{M_1 + M_2}{M_1 M_2} \pm C\sqrt{\left(\frac{M_1 + M_2}{M_1 M_2}\right)^2 - \frac{4\sin^2(Kd/2)}{M_1 M_2}},$$

to calculate the spring constant C.

(c) The crystal is exposed to light and there are several possible ways a photon can interact with the crystal. One is that the photon is absorbed creating a phonon conserving energy and momentum. Calculate the appropriate photon wave length for this process. (3p)

4. **Reflection and transmission at a potential step**

Consider an electron of energy E incident on the potential step $V(x)$,

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

where $V_0 = 4.5$ eV. Calculate the reflection coefficient R and the transmission coefficient T

a) when $E = 2.0$ eV,

b) when $E = 5.0$ eV,

c) when $E = 7.0$ eV.

(3p)

5. **Wave functions and eigenfunctions**

Consider a free particle with mass m in one dimension. The wave function of the particle at $t = 0$ is given by

$$\psi(x, t = 0) = \cos^3(kx) + \sin^3(kx).$$

(a) Show that the state function $\psi(x, 0)$ can be written as a superposition of eigenfunctions of the free-particle Hamiltonian.

(b) Determine the energy of each plane wave in the superposition.

(c) Give the wave function $\psi(x, t)$ at an arbitrary time t. (3 p)

Good Luck !