LULEA UNIVERSITY OF TECHNOLOGY Applied Physics

| Course code | F0053T |
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| Examination date | $2018-10-26$ |
| Time | $9.00-14.00$ (5 hours) |

Examination in: FASTA TILLSTÅNDETS FYsIK MED KVANTMEKANIK / Quantum Mechanics and Solid State Physics
Total number of problems: 5
Teacher on duty: Stephane Francois Tel: (49)2083, Room E159
Examiner: Hans Weber
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Allowed aids: Fysika(lia), Physics Handbook, Beta, calculator, Collection of formulae for Solid state physics and Collection of formulae for Quantum Physics.

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p .7 .5 points are required to pass the examination. Grades 3: 7.5, 4: 9.5, 5: 12.0

## 1. Crystal structure and the reciprocal lattice

Three two-dimensional structures (A, B and C) are shown in Figure 1 (you may assume that in each case the pattern is repeated to in- finity).
(a) For each structure write down a set of primitive lattice vectors, and briefly describe the primitive basis.


Figure 1: Three structures A, B and C. The markings • and $\circ$ represent different kinds of atoms.
(b) Write down the reciprocal lattice vectors for structure A.
(c) For structure A show that Bragg reflection can occur when $k-k^{\prime}=n(2 \pi / a) \hat{i}+$ $m(2 \pi / b) \hat{j}$, where $n$ and $m$ are integers. The wavevectors of the incoming and outgoing beams are $k$ and $k^{\prime}$.

## 2. Bragg scattering

Silicon (Si) and Galliumarsenid (GaAs) both have the same primitive lattice structure, fcc, with the following basises:

| Si | $(000)$, | $\left(\frac{1}{4} \frac{1}{4} \frac{1}{4}\right)$ |
| :--- | :--- | :--- |
| GaAs | $\mathrm{Ga}(000)$, | As $\left(\frac{1}{4} \frac{1}{4} \frac{1}{4}\right)$ |

Determine the the Miller indexies for the first four allowed Bragg reflections with the smallest difraction angles (glansvinkel). The atomic formfactors are not equal for any of the different atoms.

## 3. The specific heat of Gold

A measurement of the heat capacity $C_{v}$ is performed. The results are given in the table below:

| $T$ | $(\mathrm{~K})$ | 1.6 | 2.0 | 2.4 | 2.8 | 3.2 | 3.6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $C_{v}$ | $(\mathrm{~J} / \mathrm{kmol} \mathrm{K})$ | 4.18 | 6.88 | 10.7 | 15.9 | 23.0 | 31.8 |

Use these experimental results to determine the debye temperature $\Theta_{D}$ for Gold.

## 4. Misc.

a) Evaluate the commutator $\left[x^{2}, p_{x}^{2}\right]$.
b) $\mathrm{Li}^{2+}$ has the nuclear charge +3 but only one electron. How much energy does it take to excite the electron from the ground state to the level 2p? Give a numerical value in electron Volts (eV)!
c) The wave function of a hydrogen atom in an eigenstate to the Hamilton operator is:

$$
\Psi(r, \theta, \phi)=\frac{1}{81 \sqrt{6 \pi}}\left(1 / a_{\mu}\right)^{3 / 2}\left(r^{2} / a_{\mu}^{2}\right) e^{-r / 3 a_{\mu}}\left[3 \cos ^{2} \theta-1\right],
$$

where $a_{\mu}$ is the Bohr radius (with the reduced mass). Determine the quantum numbers $n, l$ och $m_{l}$.

## 5. Two dimensional Square well

A particle is placed in the potential (a 2 dimensional square well)

$$
V(x, y)=\left\{\begin{array}{cl}
0 & \text { for } \quad-\frac{a}{2} \leq x \leq \frac{a}{2} \text { and }-\frac{a}{2} \leq y \leq \frac{a}{2} \\
+\infty & \text { for } \quad x>\frac{a}{2}, x<-\frac{a}{2} \text { and } y>\frac{a}{2}, y<-\frac{a}{2}
\end{array}\right.
$$

(a) Calculate (solve the Schrödinger equation) the eigenfunctions !
(b) Write down the eigenfunctions for the ground state and one for the lowest excited states. Formulate the meaning of orthogonallity and show by explicit calculation that these two eigenfunctions are orthogonal.

## Good Luck!

