LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0053T**
Examination date	2019-06-07
Time	9.00 - 14.00 (5 hours)

Examination in: FASTA TILLSTÅNDETS FYSIK MED KVANTMEKANIK / QUANTUM MECHANICS AND SOLID STATE PHYSICS

Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysika(lia), Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE for Solid state physics and COLLECTION OF FORMULAE for Quantum Physics.

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow.
Total number of problems: **5**. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 9.5, 5: 12.0

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1. **Heat capacity**

   The figure 1 shows the temperature dependence of the heat capacity C_v for Potassium (Kalium).

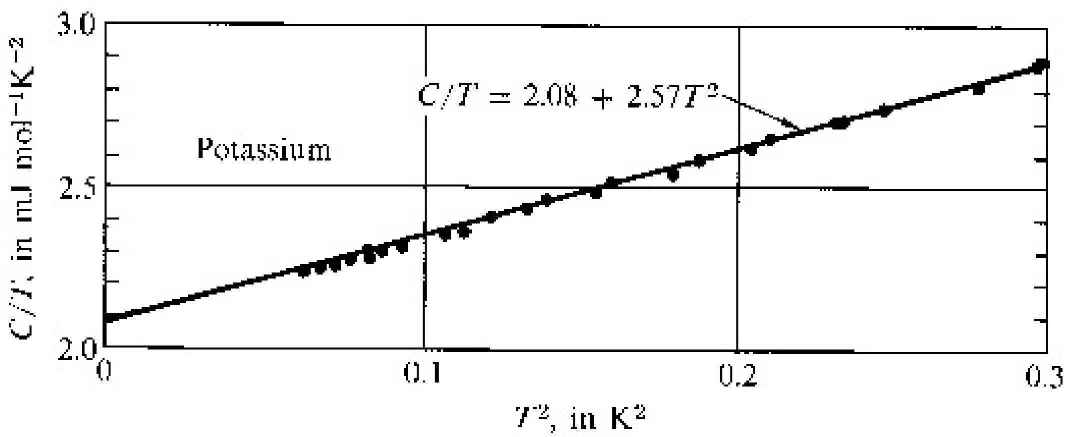

Figure 1: The specific heat of Potassium at low temperatures. The figure shows C_v/T (in mJ/(mole K^2) as a function of T^2.

   (a) What are the theories that explain a plot of C_v/T as a function of T^2 is linear.
   (b) Use the data from the figure and crystal data to determine the Debye temperature θ_D.
   (c) Use the data from the figure and crystal data to determine the efective mass of the electrons expressed in terms of the free electron mass m_0. The Fermi energy is given by $E_F = k_BT_F$.

(3p)

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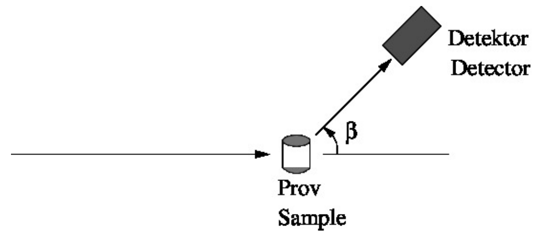

Figure 2: Principal sketch of the experiment showing the angle β

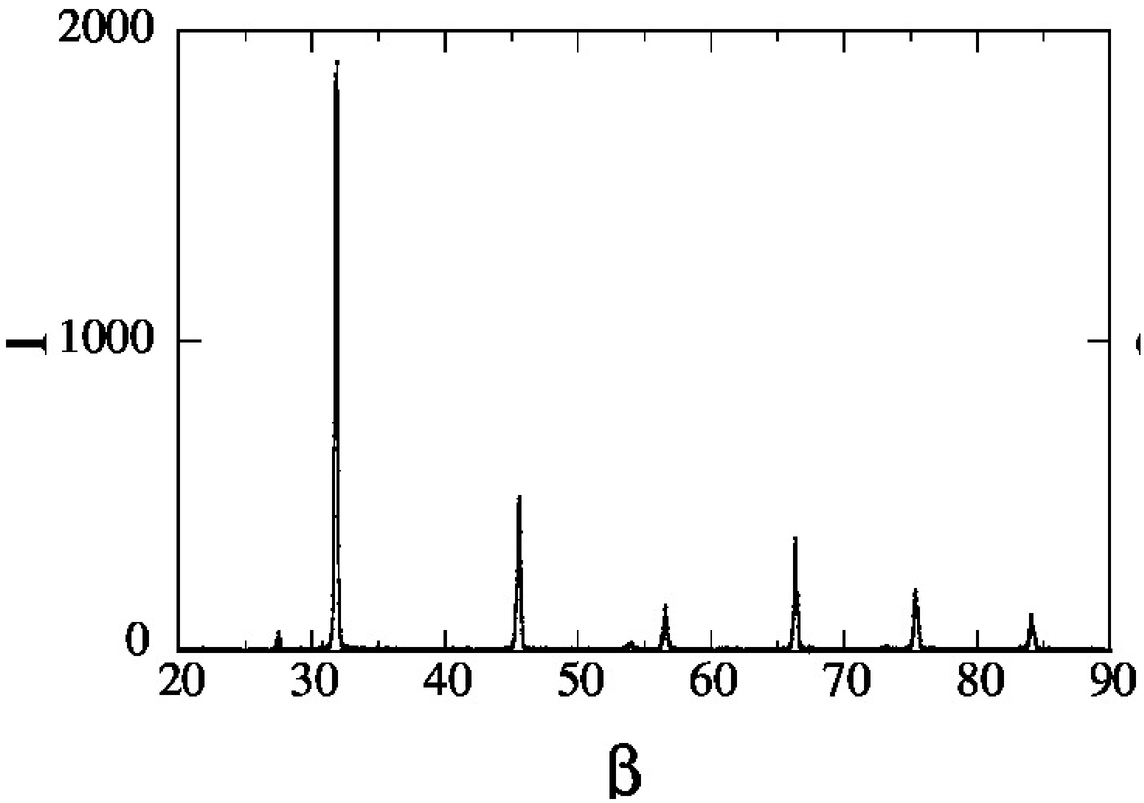

Figure 3: Bragg peaks (I for intensity) for a sample of unknown structure as a function of the angle β. **NB** There is a small peak for β between 25° and 30°.

2. **Bragg scattering.**

   The diffraction pattern of a polycrystalline di atomic ionic powder is shown in figure (3). The X-rays used is the $K_{\alpha 1}$ line from copper (Cu). The angle β (see figure (2)) can be controlled between 0° and 90°. The outcome of the experiment is presented in the figure (3), where the intensity (I) of the deflected beam is presented as a function of β.

   From the data in the figure determine the structure (sc, fcc, bcc or diamond) of the sample.

   (3p)

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3. **Reciprocal space.**

   (a) For potassium (kalium) calculate the shortest distance in reciprocal space from the origin to the surface of the Brillouin zone.

   (b) Is the Fermi sphere larger or smaller than the Brillouin zone and by how much?

(3p)

4. **Time evolution of a wave function**

A particle of mass m, which moves freely inside a one–dimensional infinite square well potential of length a, has the following initial wave function at time $t = 0$:

$$\psi(x,0) = \frac{\sqrt{13}}{\sqrt{8a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{2\sqrt{a}} \sin\left(\frac{5\pi x}{a}\right) + \frac{A}{\sqrt{a}} \sin\left(\frac{7\pi x}{a}\right)$$

where A is a real constant.

   a) Find A so that $\psi(x,0)$ is normalised.

   b) If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle $< E >$.

   c) Find the wave function $\psi(x,0)$ at any later time t.

(3p)

5. **Hydrogen like spectra**

The Institutet för rymdfysik (IRF) in Kiruna has at the moment active instruments at four different planets in our solar system. One of the instruments detects the following spectra in ultra violet light emitted from a carbon rich area.

λ (nm)	207.80	129.63	104.20	91.84

At IRF they note that the lines listed above very much appear to be like a hydrogen spectra. It is suggested the spectra originates from highly ionized carbon with only one electron left and that the lines belong to the same series ie they all have the same lower level with principal quantum number n and one may assume the upper levels are adjacent. Determine the principal quantum numbers for the levels involved in the transitions listed above.

(3p)

Good Luck !