

# 1

## Solutions to

### 1.1 Problems in Quantum Mechanics E1-E16

Last corrections 29/11 2010

$$\text{E1 } \sqrt{\frac{\pi}{a}} e^{ik_0 x} e^{-\frac{x^2}{4a}}$$

E2 QM:  $F = \frac{2E}{L}$ , CF: gives the same.

$$\text{E3 } |R|^2 = \left( \frac{\sqrt{E+V_0} - \sqrt{E}}{\sqrt{E+V_0} + \sqrt{E}} \right)^2, |T|^2 = \frac{4\sqrt{E(E+V_0)}}{(\sqrt{E+V_0} + \sqrt{E})^2}$$

$$\text{E4 } R = \frac{(k^2 + \kappa^2)(e^{\kappa a} - e^{-\kappa a})}{2ik\kappa(e^{\kappa a} - e^{-\kappa a}) + (k^2 - \kappa^2)(e^{\kappa a} - e^{-\kappa a})}$$

E5 -.

$$\text{E6 } |T|^2 \approx \exp(-2\pi\sqrt{\frac{m}{\kappa}}(-E)/\hbar)$$

E7 proof.

E8 proof.

$$\text{E9 } u(x, y, z) = \left( \frac{m\omega}{\pi\hbar} \right)^{3/4} e^{-(x^2 + y^2 + z^2)/2b^2}, \text{ where } b = \sqrt{\frac{\hbar}{m\omega}} \text{ and } u(r, \theta, \varphi) =$$

$$\text{E10 } \frac{\hbar}{i} \cos \varphi$$

$$\text{E11 Use } Y_{l,-m} = (-1)^m Y_{l,m}$$

$$\text{E12 a) } Pf(r, \theta, \varphi) = f(r, \pi - \theta, \varphi + \pi) \text{ b) } P Y_{l,l} = (-1)^l Y_{l,l} \text{ c) proof}$$

$$\text{E13 a) } \langle u_{100} | z | u_{200} \rangle = 0 \text{ b) } \langle u_{100} | z | u_{210} \rangle = \frac{1}{\sqrt{2}} \left( \frac{2}{3} \right)^6 24 \frac{a_0}{Z}$$

E14 Eigenstates are

$$\chi_{\varphi+} = \begin{pmatrix} \frac{e^{-i\varphi/2}}{\sqrt{2}} \\ \frac{e^{i\varphi/2}}{\sqrt{2}} \end{pmatrix} \quad (1.1)$$

and

$$\chi_{\varphi-} = \begin{pmatrix} \frac{e^{-i\varphi/2}}{\sqrt{2}} \\ \frac{-e^{i\varphi/2}}{\sqrt{2}} \end{pmatrix} \quad (1.2)$$

E15  $\frac{1}{2}$

E16 Eigenenergys are  $E_+ = -3.53 \cdot 10^{-26} \text{J}$  and  $E_- = +3.53 \cdot 10^{-26} \text{J}$ .

$$\chi(t) = A \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{+i 3.34 \cdot 10^{+8} t} + B \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-i 3.34 \cdot 10^{+8} t} \quad (1.3)$$

and subject to  $|A|^2 + |B|^2 = 1$ .

E17 17.6 MeV

E18 proof

E19 result for  $\Psi(x) = N \frac{\pi}{\alpha} e^{-\alpha|x|}$ , use it to make your estimates.

E20  $\phi(p) = \frac{A}{\sqrt{2\pi}} \frac{2\mu}{\mu^2 + k^2}$

E21  $A = \sqrt{\mu}$

E22 proof

E23  $\Delta E = 1.14 \text{ eV}, 1.085 \cdot 10^{-6} \text{m}$ .

E24  $n = 4 \cdot 10^7, 7.6 \cdot 10^{-8} \text{eV}$

E25 No,  $P_1 = \frac{4}{\pi^2}$  and  $P_2 = \frac{4}{\pi^2}$ .

E26 a)  $\Psi(x, t) = A \sqrt{\frac{a}{2}} \frac{1}{16} (u_5(x)e^{-iE_5t/\hbar} - 5u_3(x)e^{-iE_3t/\hbar} + 10u_1(x)e^{-iE_1t/\hbar})$  b)  $A^2 = \frac{256}{63a}$ ,  
c)  $25/126$ .

E27 Width  $\approx 1/\sqrt{\alpha}$ , gives  $E \approx (\Delta p)^2/2m = \frac{\alpha \hbar^2}{2m}$