LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	MTF107
Examination date	2003-10-27
Time	09.00 - 14.00

Solutions to Quantum Physics

1. i) $Prob = |\psi_1|^2 + |\psi_2|^2 = Prob_1 + Prob_2$

   ii) $Prob = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + (\psi_1 + \psi_2)^*(\psi_1 + \psi_2) = Prob_1 + Prob_2 + 2Re(\psi_1 + \psi_2^*)$

   The last term being due to quantum interference, giving rise to both constructive ($2Re(\psi_1 + \psi_2^*) > 0$) and destructive ($2Re(\psi_1 + \psi_2^*) < 0$) interference.

   In i) the neutron acts as a normal, but tiny, "bullet". In ii) the neutron acts as a wave, going through both slits at once!

2. a) Eigenvalues = individual results of measurements.

   b) Expectation value = average result of many measurements of an observable (on identically prepared systems).

   c) The expectation value of the operator A is $\langle A \rangle = \sum_n P_n A_n$. The eigenvalue equation being $A\psi_n = A_n\psi_n$. Here A_n = one possible eigenvalue, P_n = probability for obtaining that specific eigenvalue, $\psi_n = eigenstate$, $P_n = |c_n|^2$, where the wave function of the system is $\psi = \sum_n c_n\psi_n$.

3. As the wave function is given, we can calculate the expectation values directly as the (probability) weighted sums of the different eigenvalues. Eigenfunctions: ψ_{nlm}, where n gives the energy, l gives the total angular momentum squared according to $l(l+1)\hbar^2$, and m gives the z-component of the angular momentum, $m\hbar$.

   We begin by calculating the normalization constant, N, by the requirement

   $$\psi^*\psi = 1,$$

   giving

   $$|N|^2(4^2 + 3^2 + (-1)^2 + (\sqrt{10})^2) = 1,$$

   or

   $$N = \frac{1}{6}$$

   a) Expectation value for the energy

   $$\langle E \rangle = (\frac{4}{6})^2 E_1 + (\frac{3}{6})^2 E_2 + (\frac{-1}{6})^2 E_3 + (\frac{\sqrt{10}}{6})^2 E_4 = \frac{4}{9}E_1 + \frac{1}{4}E_2 + \frac{1}{36}E_3 + \frac{5}{18}E_4.$$

   For the hydrogen atom, the energies are $E_n \approx -13.6/n^2\ eV$, so

   $$\langle E \rangle \approx \frac{4}{9}(-13.6) + \frac{1}{4}(-3.4) + \frac{1}{36}(-1.5) + \frac{5}{18}(-0.8) \approx -7.2\,eV.$$

b) Expectation value for the total angular momentum squared

$$\langle \mathbf{L}^2 \rangle = \frac{4}{9} \cdot 0 + (\frac{9}{36} + \frac{1}{36} + \frac{10}{36}) \cdot 1(1+1)\hbar^2 = \frac{10}{9}\,\hbar^2.$$

c) Expectation value for the z-component of the angular momentum

$$\langle L_z \rangle = (\frac{16}{36} + \frac{1}{36} + \frac{10}{36}) \cdot 0 + \frac{9}{36} \cdot 1 \cdot \hbar = \frac{1}{4}\hbar.$$

4. a) $\mathbf{S}^2 \chi_{s,m_s} = s(s+1)\hbar^2 \chi_{s,m_s}$. For spin-1/2, s=1/2 giving the magnitude of spin $\sqrt{1/2(1/2+1)\hbar^2} = \frac{\sqrt{3}}{2}\hbar$

   b) Eigenstates in y-direction are $\chi_{y+} = 1/\sqrt{2}(1, i)$ for the up-state, and $\chi_{y-} = 1/\sqrt{2}(1, -i)$ for the down state. The up-state in z-direction is $\chi_+ = (1, 0) = 1/\sqrt{2}(\chi_{y+} + \chi_{y-}) = c_+ \chi_{y+} + c_- \chi_{y-}$. The probability for getting $+\hbar/2$ in y-direction is $|c_+|^2 = 1/2$. A state perfectly polarized in z-direction is thus totally unpolarized in y-direction.

5. To directly use the formula in Physics Handbook, shift x-axis by $a/2$. The wave function of the system is then $\psi = \sqrt{2/a}$ if $0 < x < a/2$, and $\psi = 0$ if $a/2 < x < a$.

   The relevant energy eigenstates of the infinite square well are $\psi_1 = \sqrt{2/a} sin(\pi x/a)$, and $\psi_2 = \sqrt{2/a} sin(2\pi x/a)$.

   The wave function can be expanded in eigenstates $\psi = \sum_n c_n \psi_n$.

   $c_1 = \int_0^a \psi_1^* \psi dx = 2/a \int_0^{a/2} sin(\pi x/a) dx = 2/\pi$

   $c_2 = \int_0^a \psi_2^* \psi dx = 2/a \int_0^{a/2} sin(2\pi x/a) dx = 2/\pi$

   The probability for ground state $= |c_1|^2 = 4/\pi^2$.

   The probability for first excited state $= |c_2|^2 = 4/\pi^2$.