LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	MTF067
Examination date	2000-12-18
Time	09.00 - 14.00

Examination in: QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Niklas Lehto — Tel: 720 85, Room E113A
Examiner: Niklas Lehto — Tel: 720 85, Room E113A
The results are put up: 22 December 2000 — on the notice-board, building E
The marking may be scrutinised: after the results have been put up

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Allowed aids: FYSIKALIA, BETA, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 18 p. 7.5 points (including bonus) is required to pass the examination.

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1. An electron is in the ground state of tritium ^3H. A β-decay instantaneously changes the atom into a helium ion $^3He^+$.
   (a) Calculate the probability that the electron is in the 2s-state ($n = 1$, $l = m = 0$) after the decay. (2p)
   (b) Calculate the probability that the electron is in a 2p-state ($n = 2$, $l = 1$) after the decay. (2p)

2. A particle is placed in a spherically symmetric potential $V(r)$. The particle is in a stationary state described by

$$\psi(\boldsymbol{r}) = \psi(x, y, z) = Nxye^{-\alpha r},$$

   where N and α are constants.
   (a) A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities. (1p)
   (b) Calculate the expectation values $\langle L^2 \rangle$ and $\langle L_z \rangle$. (1p)
   (c) The spherically symmetric potential $V(r) \to 0$ as $r \to \infty$.
   Calculate the potential. (1p)

3. A measurement of the spin component in the direction $\hat{n} = \cos\varphi\hat{x} + \sin\varphi\hat{y}$ gives the value $\hbar/2$.
   (a) Calculate the spin state corresponding to this measurement. (2p)
   (b) What would the result be of a measurement in the z-direction? (1p)

TURN PAGE!

4. For a hydrogen atom the Hamiltonian is given by

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \frac{e^2}{4\pi\epsilon_0 r}$$

(a) Give the energy expression and the number of degenerate energy levels. The electron spin is neglected. (1p)

(b) If the electron spin is taken into account, then the spin-orbit interaction will introduce a perturbation in the Hamiltonian. In a simplified model the perturbation is ϵ/r^3. Calculate the first-order energy correction. (1p)

(c) What effect will the spin-orbit interaction have on the degenerate energy levels? (1p)

5. The final destiny of a star is mainly connected to its mass. Small stars, as our sun with mass $M_\odot$, will end up as white dwarfs. Stars with a mass over $1.4M_\odot$, *the Chandrasekhar limit*, will end up as neutron stars. Finally, stars with a mass over $5.7M_\odot$ will end their lives as black holes.

All atoms in a white dwarf are fully ionized and the electrons can be seen as free electrons moving inside the star. When the mass of the dwarf is close to $1.4M_\odot$, the electrons become relativistic, *i.e.*, their energy is given by $E = pc = \hbar ck$ and not by the classical expression $E = \hbar^2 k^2/2m$.

(a) Use a box of volume $V = L^3$ to show that the Fermi energy is given by

$$E_F = \hbar c \left(\frac{3\pi^2 N}{V}\right)^{1/3},$$

for relativistic electrons. N is the number of electrons in the box. (1p)

(b) Show that the total energy of N relativistic electrons in the box is given by

$$E_{tot} = \frac{V\hbar c}{4\pi^2}\left(\frac{3\pi^2 N}{V}\right)^{4/3}.$$

(2p)

(c) The electrons give rise to a degeneracy pressure $p_{deg} = -\partial E_{tot}/\partial V$, which resists compression. At *the Chandrasekhar limit* the pressure from the relativistic electrons exactly balance the gravitational pressure in the star. The gravitational pressure is given by

$$p_g \simeq -0.69 \cdot \frac{1}{3}\left(\frac{4\pi}{3}\right)^{1/3} GM^2V^{-4/3},$$

where $G = 6.67259 \cdot 10^{-11}$ Nm^2/kg^2 is the gravitational constant. Use this to show that *the Chandrasekhar limit* is $1.4M_\odot$. Assume that there is one proton and one neutron, both of mass u, per electron. (2p)

GOOD LUCK !