LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	MTF067
Examination date Time	2001-04-18 09.00 - 14.00

Examination in: QUANTUM PHYSICS
Total number of problems: 6
Teacher on duty: Niklas Lehto — Tel: 720 85, Room E113A
Examiner: Niklas Lehto — Tel: 720 85, Room E113A
The results are put up: 27 April 2001 — on the notice-board, building E
The marking may be scrutinised: after the results have been put up

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Allowed aids: FYSIKALIA, BETA, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 18 p. 7.5 points is required to pass the examination.

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1. Five identical particles are placed in the potential

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 \leq x \leq a \\ +\infty & \text{for} \quad x > a,\ x < 0. \end{cases}$$

   There is no interaction between the particles other than the fact that they are identical.

   (a) Calculate ground state energy if the particles are identical spin 0 bosons. (1,5p)

   (b) Calculate ground state energy if the particles are identical spin $\frac{1}{2}$ fermions. (1,5p)

2. Calculate the commutator $[L_z, sin(\phi)]$, where ϕ is the spherical angle $\phi = \arctan(y/x) + n\pi$. (3 p)

3. Consider a free particle with mass m in one dimension. The wave function of the particle at $t = 0$ is given by

$$\psi(x, 0) = \cos^3 kx.$$

   (a) Show that the state function $\psi(x, 0)$ can be written as a superposition of eigenfunctions of the free-particle Hamiltonian. (1p)

   (b) Determine the energy of each plane wave in the superposition. (1p)

   (c) Give the wave function $\psi(x, t)$ at an arbitrary time t. (1p)

TURN PAGE!

4. Particles with energy E and mass m are coming from $-\infty$ towards the potential step:

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0, \end{cases}$$

where $E > V_0 > 0$.

(a) Determine the wave functions in the two regions. (1,5p)

(b) Calculate the number of transmitted particles per time unit, if the incoming particle current is N_0 particles per time unit. (1,5p)

5. A measurement of the spin component in the direction $\hat{n} = \cos\varphi \hat{x} + \sin\varphi \hat{y}$ gives the value $\hbar/2$.

(a) Calculate the spin state corresponding to this measurement. (2p)

(b) What would the result be of a measurement in the z-direction? (1p)

6. An electron is in the ground state of tritium ^3H. A β-decay instantaneously changes the atom into a helium ion $^3He^+$.

(a) Calculate the probability that the electron is in the 2s-state ($n = 2$, $l = m = 0$) after the decay. (2 p)

(b) Calculate the probability that the electron is in a 2p-state ($n = 2$, $l = 1$) after the decay. (1p)

GOOD LUCK !