LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

| Course code | MTF067 |
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| Examination date | 2001-08-29 |
| Time | 09.00 - 14.00 |

Examination in: QUANTUM PHYSICS
Total number of problems: 6
Teacher on duty: Niklas Lehto    Tel: 720 85, Room E113A
Examiner: Niklas Lehto    Tel: 720 85, Room E113A
The results are put up: 7 September 2001    on the notice-board, building E
The marking may be scrutinised: after the results have been put up

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Allowed aids: FYSIKALIA, BETA, calculator, COLLECTION OF FORMULAE

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Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 18 p. 7.5 points is required to pass the examination.

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1. A particle is placed in the potential

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 \leq x \leq a \\ +\infty & \text{for} \quad x > a,\ x < 0. \end{cases}$$

   (a) Determine the eigenfunctions and energy eigenvalues of the particle. (1p)

   (b) At $t = 0$ the wave function of the particle is

$$\psi(x, 0) = Nx(x - a),$$

   where $N$ is a normalization factor. What is the probability that a measurement of the energy, $E$, at $t = 0$ will give a result

$$E \leq \frac{3\hbar^2\pi^2}{ma^2}.$$

   (2p)

2. A particle is placed in a spherically symmetric potential $V(r)$. The particle is in a stationary state described by

$$\psi(\boldsymbol{r}) = \psi(x, y, z) = Nxye^{-\alpha r},$$

   where $N$ and $\alpha$ are constants.

   (a) A measurement of $L^2$ and $L_z$ is done on the system. Calculate the possible values and their probabilities. (1p)

   (b) Calculate the expectation values $\langle L^2 \rangle$ and $\langle L_z \rangle$. (1p)

   (c) The spherically symmetric potential $V(r) \to 0$ as $r \to \infty$. Calculate the potential. (1p)

TURN PAGE!

3. Calculate the commutator $[L_z, sin(\phi)]$, where $\phi$ is the spherical angle $\phi = \arctan(y/x) + n\pi$. (3p)

4. For a hydrogen atom the Hamiltonian is given by

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \frac{e^2}{4\pi\epsilon_0 r}$$

   (a) Give the energy expression and the number of degenerate energy levels. The electron spin is neglected. (1p)
   (b) If the electron spin is taken into account, then the spin-orbit interaction will introduce a perturbation in the Hamiltonian. In a simplified model the perturbation is $\epsilon/r^3$. Calculate the first-order energy correction. (1p)
   (c) What effect will the spin-orbit interaction have on the degenerate energy levels? (1p)

5. Particles with energy $E$ and mass $m$ are coming from $-\infty$ towards the potential step:

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0, \end{cases}$$

   where $E > V_0 > 0$.

   (a) Determine the wave functions in the two regions. (1,5p)
   (b) Calculate the number of transmitted particles per time unit, if the incoming particle current is $N_0$ particles per time unit. (1,5p)

6. A measurement of the spin component in the direction $\hat{n} = \cos\varphi\hat{x} + \sin\varphi\hat{y}$ gives the value $\hbar/2$.

   (a) Calculate the spin state corresponding to this measurement. (2p)
   (b) What would the result be of a subsequent measurement in the $z$-direction? (1p)

GOOD LUCK !