LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	MTF067
Examination date	2002-12-20
Time	09.00 - 14.00

Examination in: QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Johan Hansson — Tel: 491072, Room E102a
Examiner: Johan Hansson — Tel: 491072, Room E102a
The results are announced: January 17, 2003 — on the notice-board, building E
The grading may be scrutinised: after the results have been announced

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Allowed aids: FYSIKALIA, PHYSICS HANDBOOK, *ANY* mathematical handbook (for example BETA), calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. The maximum number of points is 15 p. 7 points is required to pass the examination (grade 3), 10.5 points for grade 4, 13 points for grade 5.

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1. a) Write down the general eigenvalue equation for the energy of a quantum mechanical system. (1p)

   b) What is the *mathematical criterion* on an operator for its eigenvalues to be *observables*? (1p)

   c) Give an example of a known quantum mechanical operator which, by itself, cannot give an eigenvalue equation. (1p)

2. An electron in a hydrogen atom is in the state described by the wave function

$$\psi(\mathbf{r}) = N[4\psi_{100}(\mathbf{r}) + 3i\psi_{211}(\mathbf{r}) - i\psi_{310}(\mathbf{r}) + \sqrt{10}\psi_{410}(\mathbf{r})].$$

   a) What is the expectation value of the energy?

   b) What is the expectation value of $\mathbf{L}^2$?

   c) What is the expectation value of L_z? (3p)

3. The spin-part of a spin-1/2 quantum mechanical system is given by

$$\chi = 2|\uparrow\rangle + i|\downarrow\rangle.$$

   a) What are the possible eigenvalues of $\mathbf{S}^2$ and S_z? Are they simultaneously measurable? (Show mathematically why/why not.) (1p)

   b) Calculate the probabilities for obtaining these eigenvalues. (2p)

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4. Calculate $\langle x \rangle$, $\langle x^2 \rangle$ and $\langle x^3 \rangle$ for a one-dimensional quantum mechanical harmonic oscillator in a given state n.

(3p)

5. The Hamiltonian (in units of eV) for a given axially symmetric quantum rotator is

$$H = \frac{L_x^2 + L_y^2}{2\hbar^2} + \frac{L_z^2}{3\hbar^2}.$$

What are its possible energies? (3p)

GOOD LUCK !