

Course code	MTF107 / MTF067
Examination date	2008-08-30
Time	09.00 - 14.00

Examination in: **QUANTUM MECHANICS**

Total number of problems: 5

Teacher on duty: Hans Weber

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The results are put up:

22 September 2008 on the notice-board, building E

The marking may be scrutinised:

after the results have been put up

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

1. Hydrogen atom

- a) Determine for a 2p-electron the most probable distance from the nucleus of the hydrogen atom. (Do the calculations for the radial part only)
- b) Which of the following ten hydrogen states have the same energy (no external fields are applied). 3s with $m_l = 0$, 3p with $m_l = 1$, 4d with $m_l = 1$, 3p with $m_l = -1$, 5d with $m_l = 1$, 4p with $m_l = 0$, 5p with $m_l = -1$, 4p with $m_l = -1$, 5s with $m_l = 0$, 3p with $m_l = 0$.

(3p)

2. Three-dimensional box well

A particle is placed in the potential (a 3 dimensional box well)

$$V(x, y, z) = \begin{cases} 0 & \text{for } 0 \leq x \leq a \text{ and } 0 \leq y \leq a \text{ and } 0 \leq z \leq \frac{a}{2} \\ +\infty & \text{for } x > a \text{ or } x < 0 \text{ or } y > a \text{ or } y < 0 \text{ or } z > \frac{a}{2} \text{ or } z < 0. \end{cases}$$

- (a) Calculate (solve the Schrödinger equation) the eigenfunctions ?
- (b) What are the 6 lowest eigenenergies ?
- (c) What are the degeneracies of the states associated to these 6 lowest eigenenergies ?

(3p)

3. Spin

A spin 1/2 particle described by the unnormalised spinor χ

$$\chi = A \begin{pmatrix} 2 + 5i \\ 3 - i \end{pmatrix}.$$

- Evaluate the expectation values of the three Cartesian components ($\langle S_x \rangle$, $\langle S_y \rangle$, $\langle S_z \rangle$).
- For a measurement of spin along the x direction what are the possible outcomes of this experiment and their probabilities?

(3p)

4. Angular momentum and r in Hydrogen

An electron bound in a hydrogen atom is described by the following state:

$$\psi(\mathbf{r}) = \psi(x, y, z) = Nxz e^{-\sqrt{x^2+y^2+z^2}/3a_0},$$

where a_0 is the Bohr radius and N is a constant (normalisation).

- A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities.
- Calculate the expectation value of the electrons distance $\langle r \rangle$ from the nucleus.

(3p)

5. Eigenfunctions and uncertainty

An electron confined in a quantum well has four discrete energy levels $E_1 = 0.27$ eV, $E_2 = 1.08$ eV, $E_3 = 3.65$ eV, $E_4 = 4.06$ eV. It is in a state in which the probabilities associated with these energies are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{16}$ and $\frac{1}{16}$ respectively.

- Find the expectation value of its energy $\langle \hat{H} \rangle$ and the corresponding uncertainty $\Delta \hat{H}$.
- Obtain an expression for the wave function $\Psi(z)$ describing the state of the particle in terms of its energy eigenfunctions $\psi_n(z)$ at time $t = 0$. Why is the expression not unique? Write down two different wave functions corresponding to the same values of $\langle \hat{H} \rangle$ and $\Delta \hat{H}$ that you found in (a).

(3 p)

GOOD LUCK !