LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0047T/MTF107
Examination date	2010-01-15
Time	09.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: 492088, Room B253
Examiner: Hans Weber Tel: 492088 or 0708-592088, Room B253

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Parity operator**

   The parity operator $\hat{\Pi}$, acts on a function $\Psi(x)$ in the following way $\hat{\Pi}\Psi(x) = \Psi(-x) = \lambda\Psi(x)$ where the equality is valid only if the function Ψ is an eigenfunction to the operator $\hat{\Pi}$, the eigenvalue λ can take two possible values ± 1.

   (a) Show that the following functions are eigenfunctions of the parity operator and find the corresponding eigenvalue in each case.

   i. $\Psi_1(x) = C\left(\sin(\frac{\pi x}{L}) + \sin(\frac{3\pi x}{L})\right)$

   ii. $\Psi_2(x,y,z) = Ce^{-a\sqrt{x^2+y^2+z^2}}$

   iii. $\Psi_3(r,\theta,\phi) = Cf(r)\left(\cos(\theta) + cos^3(\theta)\right)\ e^{i\phi}$

   (b) If $\psi_+(x,y,z)$ and $\psi_-(x,y,z)$ are eigenfunctions of $\hat{\Pi}$ corresponding to eigenvalues +1 and -1 respectively.

   i. Is the function $\Psi = 2\psi_+(x,y,z) + 3\psi_-(x,y,z)$ an eigenfunction of the parity operator?

   ii. Show that it is an eigenfunction of $\hat{\Pi}^2$, and find the eigenvalue.

   iii. Do the functions e^{-ikx} and e^{ikx} have parity? If yes what are their eigenvalues. If no form linear combinations that have a definite parity and what are their eigenvalues.

   (3 p)

2. **Wave functions and eigenfunctions**

   Consider a free particle with mass m in one dimension. The wave function of the particle at $t = 0$ is given by

   $$\psi(x, t = 0) = \cos^3(kx) + \sin^5(kx).$$

   (a) Show that the state function $\psi(x, 0)$ can be written as a superposition of eigenfunctions of the free-particle Hamiltonian.

   (b) Determine the energy of each plane wave in the superposition.

   (c) Give the wave function $\psi(x,t)$ at an arbitrary time t. (3 p)

TURN PAGE!

3. **Hydrogen atom**

Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

$$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{15}} \left(2\psi_{100}(\mathbf{r}) - 3\psi_{200}(\mathbf{r}) + \psi_{310}(\mathbf{r}) + \psi_{322}(\mathbf{r})\right)$$

(a) Is this wave function an eigenfunction of the parity operator $\hat{\Pi}$?

(b) What is the probability of finding the system in the ground state? In the state (200)? In the state (310)? In the state (322)? In any other state?

(c) What is the expectation value of the energy; of the operator $\mathbf{L}^2$; of the the operator L_z.

(3 p)

4. **Perturbation calculation**

A particle of mass $m = m_e$ is in a infinite square well potential of width $a = 10.0$Å. Due to imperfections the potential takes the following form:

$$V(x) = \begin{cases} \infty & \text{for} \quad x < 0, \\ \epsilon = 0.11\text{eV} & \text{for} \quad 0 < x < \frac{a}{2}, \\ 0 & \text{for} \quad \frac{a}{2} < x < a, \\ \infty & \text{for} \quad x > a \end{cases}$$

(a) Use perturbation theory to calculate to lowest order the corrections to the energy of the ground state and the first excited state. Give both the unperturbed levels and the perturbed ones (in eV).

(b) Now if $\epsilon = 1.08$ eV, would the perturbation calculation yield a good value for the perturbed levels? Motivate your answer! (3 p)

5. **Molecular spectra**

The observed rotational spectrum of the hydrogen chloride molecule consists of a set of equally spaced lines produced by the emission (or absorption) of electric dipole radiation. The spacing is 20.68 cm^{-1}.

(a) Calculate the moment of inertia of the HCl molecule.

(b) What are the energies of its four lowest rotational energy levels?

Assuming that the moment of inertia of a diatomic molecule is of the form $I = mr^2$, estimate the mean separation r of the atom in the molecules. (Here $m = m_H m_{Cl}/(m_H + m_{Cl})$ is the reduced mass of the molecule. Use the mass of the isotope ^{35}Cl: $m_{Cl} = 35 m_H$. (3 p)

GOOD LUCK !