LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0047T/MTF107
Examination date	2010-03-16
Time	09.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Nils Almqvist Tel: 492291 or 0705-898710, Room B254
Examiner: Hans Weber Tel: 492088 or 0708-592088, Room B253

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

---

1. **Two dimensional Square well**

   A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions !

   (b) The Hamiltonian commutes with the parity operator P, $P\Psi(x,y) = \Psi(-x,-y) = \lambda\Psi(x,y)$ where the eigenvalue λ can take two possible values ± 1.
   Write down the eigenstates corresponding to the four lowest **energies** in such a way that they are also eigenfunctions of the parity operator P. What is the parity of these states?

(3p)

2. **Measurement of spin**

   A measurement of the spin component in the direction $\hat{n} = \cos\varphi\hat{x} + \sin\varphi\hat{y}$ gives the value $\hbar/2$.

   (a) Calculate the spin state corresponding to this measurement.

   (b) What would the result be of a measurement in the z-direction?

(3p)

TURN PAGE!

3. **Eigenfunctions and uncertainty**

An electron confined in a quantum well has five discrete energy levels $E_1 = 0.31$ eV, $E_2 = 0.97$ eV, $E_3 = 1.81$ eV, $E_4 = 3.35$ eV, $E_5 = 4.08$ eV. It is in a state in which the probabilities associated with these energies are $\frac{1}{2}$, $\frac{2}{12}$, $\frac{1}{12}$, $\frac{3}{16}$ and $\frac{1}{16}$ respectively.

(a) Find the expectation value of its energy $\langle \hat{H} \rangle$ and the corresponding uncertainty $\Delta \hat{H}$.

(b) Obtain an expression for the wave function $\Psi(z)$ describing the state of the particle in terms of its energy eigenfunctions $\psi_n(z)$ at time $t = 0$. Why is the expression not unique? Write down two different wave functions corresponding to the same values of $\langle \hat{H} \rangle$ and $\Delta \hat{H}$ that you found in (a).

(c) Assume the potential is the infinite square well of width L, and you would have calculated $\langle \hat{H} \rangle$ in some way. If one adiabatically changes L to $L/2$ by how much would $\langle \hat{H} \rangle$ change? Adiabatically means we are not inducing transitions between levels in the system.

(3 p)

4. **Harmonic oscillator.**

A particle of mass m is confined to a one dimensional potential of the following form:

$$V(x) = \frac{1}{2}k(x - x_0)^2.$$

Show that the wave function of the ground state can be written as Ae^{ax^2+bx} and determine the constants a and b.

What are the possible values the energy of the particle can take? (3p)

5. **Angular momentum and r in Hydrogen**

An electron bound in a hydrogen atom is described by the following state:

$$\psi(\boldsymbol{r}) = \psi(x, y, z) = Nxze^{-\sqrt{x^2+y^2+z^2}/3a_0},$$

where a_0 is the Bohr radius and N is a constant (normalisation).

(a) A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities.

(b) Calculate the expectation value of the electrons distance $< r >$ from the nucleus.

(3p)

GOOD LUCK !