LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0047T/MTF107
Examination date	2010-08-18
Time	09.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: 492088, Room B253
Examiner: Hans Weber Tel: 492088 or 0708-592088, Room B253

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Perturbation calculation**

   A particle of mass $m = m_e$ is in a infinite square well potential of width $a = 4.0$Å. Due to imperfections the potential takes the following form:

$$V(x) = \begin{cases} \infty & \text{for} \quad x < 0, \\ \epsilon = 0.63\text{eV} & \text{for} \quad 0 < x < \frac{a}{2}, \\ 0 & \text{for} \quad \frac{a}{2} < x < a, \\ \infty & \text{for} \quad x > a \end{cases}$$

   Consider a transition from the lowest excited state to the groundstate: Use perturbation theory to calculate to lowest order the corrections to the transition energy. Please answer in eV. (3 p)

2. **Reflection and transmission at a potential step**

   Consider an electron of energy E incident on the potential step $V(x)$,

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

   where $V_0 = 5.0$ eV. Calculate the reflection coefficient R and the transmission coefficient T

   a) when $E = 2.5$ eV,
   b) when $E = 5.0$ eV,
   c) when $E = 7.5$ eV.

   (3p)

TURN PAGE!

3. **Quantum states of Tritium and Helium**

An electron is in the ground state of tritium ^3H. A β-decay instantaneously changes the atom into a helium ion $^3He^+$. The β particle (=high energy electron) leaves the helium ion and is no longer to be taken inte consideration. The helium ion that is left behind has one single electron bound to it.

(a) Calculate the probability that the electron (bound to helium ion) is in the 2s-state ($n = 2$, $l = m = 0$) after the decay.

(b) Calculate the probability that the electron is in a 2p-state ($n = 2$, $l = 1$) after the decay.

(c) Calculate the probability that the electron is in a 1s-state ($n = 1$, $l = m = 0$) after the decay.

(d) Is it possible for the electron to recieve the quantum numbers ($n = 1$, $l = 1$) after the decay?

(3p)

4. **Harmonic oscillator solution**

A particle is confined to a harmonic oscillator potential.

a) Show that the two functions $\psi^+(\xi) = A\xi e^{+\xi^2/2}$ and $\psi^-(\xi) = B\xi e^{-\xi^2/2}$ are eigenfunctions of the linear harmonic oscillator equation (in dimensionless form):

$$\frac{d^2\psi(\xi)}{d\xi^2} + (\lambda - \ \xi^2)\psi(\xi) = 0$$

and determine the two eigenvalues λ^+ (for $\psi^+(\xi)$) and λ^- (for $\psi^-(\xi)$). Where $\xi = \sqrt{\frac{m\omega}{\hbar}}x$ and $\lambda = \frac{2E}{\hbar\omega}$, A and B are constants.

b) Are the eigenfunctions physical acceptable (both/one or none)? Motivate!

(3p)

5. **Time evolution of solution**

A particle of mass m, which moves freely inside a one–dimensional infinite square well potential of length a, has the following initial wave function at time $t = 0$:

$$\psi(x, 0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$$

where A is a real constant.

a) Find A so that $\psi(x, 0)$ is normalised.

b) If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle $< E >$.

c) Find the wave function $\psi(x, 0)$ at any later time t.

(3p)

GOOD LUCK !