LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0047T/MTF107
Examination date Time	2011-01-11 09.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber   Tel: 492088, Room E304
Examiner: Hans Weber   Tel: 492088 or 0708-592088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Perturbation calculation**

   An electron of mass $m = m_e$ is in a infinite square well potential of width $a = 2.0$Å. Due to imperfections the potential takes the following form:

$$V(x) = \begin{cases} \infty & \text{for} \quad x < 0, \\ \epsilon = 0.47\text{eV} & \text{for} \quad 0 < x < \frac{a}{2}, \\ 0 & \text{for} \quad \frac{a}{2} < x < a, \\ \infty & \text{for} \quad x > a \end{cases}$$

   Consider a transition from the lowest excited state to the ground state: Use perturbation theory to calculate to lowest order the corrections to the transition energy. Please answer in eV. (3 p)

2. **Operators and eigenfunctions**

   Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

   (a) $\psi(t) = \sin \omega t$ and $\hat{A} = i\hbar \frac{\partial^2}{\partial t^2}$.

   (b) $\psi(z) = C(1 + z^2)$ and $\hat{A} = -i\hbar \frac{\partial}{\partial z}$.

   (c) $\psi(z) = C_1 e^{ikz} + C_2 e^{-ikz}$ and $\hat{A} = -\hbar^2 \frac{\partial^2}{\partial z^2}$.

   (d) $\psi(z) = Ce^{-3z}$ and $\hat{A} = -i\frac{\hbar}{2}\frac{\partial}{\partial z}$.

   (e) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (f) $\psi(z) = Ce^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (3 p)

TURN PAGE!

3. **Measurement of spin**

A measurement of the spin component in the direction $\hat{n} = \hat{x}\cos(\varphi) + \hat{y}\sin(\varphi)$ gives the value $\hbar/2$, where $\hat{x}$ and $\hat{y}$ are unit vectors.

(a) Calculate the spin state corresponding to this measurement.

(b) What would the result be of a measurement in the z-direction?

(c) If one would after the measurement in b) make a new measurement in the direction $\hat{n}$ what would the probability be to get the value $\hbar/2$ again? Motivate !

(3p)

4. **Quantum states of Tritium and Helium**

An electron is in the ground state of tritium ^3H. A β-decay instantaneously changes the atom into a helium ion $^3He^+$. The β particle (=high energy electron) leaves the helium ion and is no longer to be taken into consideration. The helium ion that is left behind has one single electron bound to it.

(a) Calculate the probability that the electron (bound to helium ion) is in the 2s-state ($n = 2$, $l = m = 0$) after the decay.

(b) Calculate the probability that the electron is in a 2p-state ($n = 2$, $l = 1$) after the decay.

(c) Calculate the probability that the electron is in a 1s-state ($n = 1$, $l = m = 0$) after the decay.

(d) Is it possible for the electron to receive the quantum numbers ($n = 1$, $l = 1$) after the decay?

(3p)

5. **Parity operator**

The parity operator $\hat{\Pi}$, acts on a function $\Psi(x)$ in the following way $\hat{\Pi}\Psi(x) = \Psi(-x) = \lambda\Psi(x)$ where the equality is valid only if the function Ψ is an eigenfunction to the operator $\hat{\Pi}$, the eigenvalue λ can take two possible values ± 1.

(a) Show that the following functions are eigenfunctions of the parity operator and find the corresponding eigenvalue in each case.

i. $\Psi_1(x) = C\left(\sin(\frac{\pi x}{L}) + \sin(\frac{3\pi x}{L})\right)$

ii. $\Psi_2(x, y, z) = Ce^{-a\sqrt{x^2+y^2+z^2}}$

iii. $\Psi_3(r, \theta, \phi) = Cf(r)\left(\cos(\theta) + cos^3(\theta)\right)\; e^{i\phi}$

(b) If $\psi_+(x, y, z)$ and $\psi_-(x, y, z)$ are eigenfunctions of $\hat{\Pi}$ corresponding to eigenvalues +1 and -1 respectively.

i. Is the function $\Psi = 2\psi_+(x, y, z) + 3\psi_-(x, y, z)$ an eigenfunction of the parity operator?

ii. Show that it is an eigenfunction of $\hat{\Pi}^2$, and find the eigenvalue.

iii. Do the functions e^{-ikx} and e^{ikx} have parity? If yes what are their eigenvalues. If no form linear combinations that have a definite parity and what are their eigenvalues.

(3 p)

GOOD LUCK !