LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0047T/MTF107
Examination date	2011-08-27
Time	09.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: 492088, Room E304
Examiner: Hans Weber Tel: 492088 or 0708-592088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Molecular spectra**

   The observed rotational spectrum of the hydrogen chloride molecule consists of a set of equally spaced lines produced by the emission (or absorption) of electric dipole radiation. The spacing is 20.68 cm^{-1}.

   (a) Calculate the moment of inertia of the HCl molecule.

   (b) What are the energies of its four lowest rotational energy levels?

   Assuming that the moment of inertia of a diatomic molecule is of the form $I = mr^2$, estimate the mean separation r of the atom in the molecules. (Here $m = m_H m_{Cl}/(m_H + m_{Cl})$ is the reduced mass of the molecule. Use the mass of the isotope ^{35}Cl: $m_{Cl} = 35 m_H$. (3 p)

2. **Three–dimensional box well**

   A particle is placed in the potential (a 3 dimensional box well)

$$V(x,y,z) = \begin{cases} 0 & \text{for} \quad 0 \le x \le a \text{ and } 0 \le y \le a \text{ and } 0 \le z \le \frac{\mathbf{a}}{\mathbf{2}} \\ +\infty & \text{for} \quad x > a \text{ or } x < 0 \text{ or } y > a \text{ or } y < 0 \text{ or } z > \frac{\mathbf{a}}{\mathbf{2}} \text{ or } z < 0. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions ?

   (b) What are the 7 lowest eigenenergies ?

   (c) What are the degeneracies of the states associated to these 7 lowest eigenenergies ?

   (3p)

3. **Quantum rotator**

The Hamiltonian (in units of eV) for a given axially symmetric quantum rotator is

$$H = \frac{L_x^2 + L_y^2}{2\hbar^2} + \frac{L_z^2}{3\hbar^2}$$

What are the possible energies? (3p)

4. **Angular momentum and r in Hydrogen**

An electron bound in a hydrogen atom is described by the following state:

$$\psi(\boldsymbol{r}) = \psi(x, y, z) = Nxze^{-\sqrt{x^2+y^2+z^2}/3a_0},$$

where a_0 is the Bohr radius and N is a constant (normalisation).

(a) A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities.

(b) Calculate the expectation value of the electrons distance $< r >$ from the nucleus.

(3p)

5. **Harmonic oscillator solution**

A particle is confined to a harmonic oscillator potential.

a) Show that the two functions $\psi^+(\xi) = A\xi e^{+\xi^2/2}$ and $\psi^-(\xi) = B\xi e^{-\xi^2/2}$ are eigenfunctions of the linear harmonic oscillator equation (in dimensionless form):

$$\frac{d^2\psi(\xi)}{d\xi^2} + (\lambda - \ \xi^2)\psi(\xi) = 0$$

and determine the two eigenvalues λ^+ (for $\psi^+(\xi)$) and λ^- (for $\psi^-(\xi)$). Where $\xi = \sqrt{\frac{m\omega}{\hbar}}x$ and $\lambda = \frac{2E}{\hbar\omega}$, A and B are constants.

b) Are the eigenfunctions physical acceptable (both/one or none)? Motivate!

(3p)

GOOD LUCK !