LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0047T/MTF107
Examination date Time	2012-01-10 09.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E304
Examiner: Hans Weber Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Eigenfunctions and uncertainty**

   An electron confined in a quantum well has five discrete energy levels $E_1 = 0.25$ eV, $E_2 = 0.95$ eV, $E_3 = 2.12$ eV, $E_4 = 3.23$ eV, $E_5 = 4.79$ eV. It is in a state in which the probabilities associated with these energies are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{24}$ and $\frac{1}{24}$ respectively.

   (a) Find the expectation value of its energy $\langle \hat{H} \rangle$ and the corresponding uncertainty $\Delta \hat{H}$.

   (b) Obtain an expression for the wave function $\Psi(x)$ describing the state of the particle in terms of its energy eigenfunctions $\psi_n(x)$ at time $t = 0$. Why is the expression not unique? Write down two different wave functions corresponding to the same values of $\langle \hat{H} \rangle$ and $\Delta \hat{H}$ that you found in (a).

   (c) Assume the potential is the infinite square well of width L, and you would have calculated $\langle \hat{H} \rangle$ in some way. If one adiabatically would increase L to $2L$ by how much would $\langle \hat{H} \rangle$ change? Adiabatically means we are not inducing transitions between levels in the system.

   (3 p)

2. **Early model of the nucleus of an atom**

   In the early days it was thought that the nucleus of an atom also contained electrons as they where emitted in a β decay. Today we know this is not the case. Make a reasonable estimate of the kinetic energy of an electron confined to a box of an approximate size of 1 fm (the size of a nucleus) ? Give your answer in eV !

   (*Hint*, start with a choice of an appropriate model for your calculation. Motivate choice.)

   (3 p)

TURN PAGE!

3. **Hydrogen like spectra**

The Institutet för rymdfysik (IRF) in Kiruna has at the moment active instruments at four different planets in our solar system. One of the instruments detects the following spectra in ultra violet light emitted from a carbon rich area.

λ (nm)	207.80	129.63	104.20	91.84

At IRF they note that the lines listed above very much appear to be like a hydrogen spectra. It is suggested the spectra originates from highly ionized carbon with only one electron left and that the lines belong to the same series ie they all have the same lower level with principal quantum number n and one may assume the upper levels are adjacent. Determine the principal quantum numbers for the levels involved in the transitions listed above.

(3p)

4. **Two dimensional Square well and parity**

A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

(a) Calculate (solve the Schrödinger equation !) the eigenfunctions !

(b) The Hamiltonian commutes with the parity operator P, $P\Psi(x,y) = \Psi(-x,-y) = \lambda\Psi(x,y)$ where the eigenvalue λ can take two possible values ± 1.

Write down the eigenstates corresponding to the four lowest **energies** in such a way that they are also eigenfunctions of the parity operator P. What is the parity of these states?

(3p)

5. **Perturbation calculation**

Consider a one–dimensional harmonic oscillator of mass m and angular frequency ω with the Hamiltonian:

$$H^0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2$$

Due to the surrounding of the oscillator adding an anharmonic term $H^1 = Ax^4$ to the unperturbed H^0 would more correctly describe the system (A is a constant).

Calculate the energy difference between the first excited level and the ground state of the perturbed system.

(*Hint*, integrate by parts. For the unperturbed system the energy difference would be $\hbar\omega$)

(3 p)

GOOD LUCK !