LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0047T/MTF107
Examination date	2012-03-13
Time	09.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E304
Examiner: Hans Weber Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Reflection and transmission at a potential step**

   Consider an electron of energy E incident on the potential step $V(x)$,

   $$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

   where $V_0 = 3.5$ eV. Calculate the reflection coefficient R and the transmission coefficient T

   a) when $E = 2.0$ eV,

   b) when $E = 5.0$ eV,

   c) when $E = 7.0$ eV.

   (3p)

2. **Time evolution of solution**

   A particle of mass m, which moves freely inside a one–dimensional infinite square well potential of length a, has the following initial wave function at time $t = 0$:

   $$\psi(x,0) = \frac{A}{\sqrt{2a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{2\sqrt{a}} \sin\left(\frac{5\pi x}{a}\right) + \frac{1}{\sqrt{8a}} \sin\left(\frac{7\pi x}{a}\right)$$

   where A is a real constant.

   a) Find A so that $\psi(x,0)$ is normalised.

   b) If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle $< E >$.

   c) Find the wave function $\psi(x,0)$ at any later time t.

   (3p)

TURN PAGE!

3. **Operators and eigenfunctions**

   Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

   (a) $\psi(t) = \sin \omega t$ and $\hat{A} = i\hbar \frac{\partial^2}{\partial t^2}$.

   (b) $\psi(z) = C(1 + z^2)$ and $\hat{A} = -i\hbar \frac{\partial}{\partial z}$.

   (c) $\psi(z) = C_1 e^{ikz} + C_2 e^{-ikz}$ and $\hat{A} = -\hbar^2 \frac{\partial^2}{\partial z^2}$.

   (d) $\psi(z) = Ce^{-3z}$ and $\hat{A} = -i\frac{\hbar}{2} \frac{\partial}{\partial z}$.

   (e) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (f) $\psi(z) = Ce^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (3 p)

4. **Measurement of spin**

   A spin $\frac{1}{2}$ particle is prepared to be in an eigenstate to S_z with eigenvalue $+\frac{1}{2}\hbar$. A subsequent measurement of the spin in the direction $\hat{n} = \sin(\varphi)\hat{e}_y + \cos(\varphi)\hat{e}_z$ is made. The value of φ is $\pi/4$.

   (a) What is the probability to get the value $+\hbar/2$ and $-\hbar/2$ in this new direction $\hat{n}$?

   (b) What would the result (eigenvalue and probability) be of a subsequent measurement in the z-direction of the $+\hbar/2$ state in a) ?

   (3p)

5. **Molecular spectra**

   In the rotational fine structure spectra of $^1H^{35}Cl$ the following spectral lines where detected 2824,0 cm^{-1}; 2844,6 cm^{-1}; 2865,3 cm^{-1}; 2906,7 cm^{-1}; 2927,4 cm^{-1}; 2948,0 cm^{-1} och 2968,7 cm^{-1}.

   a) There seems to be a line missing, explain why.

   b) Using the data given calculate the distance between the atoms of the molecule.

   (3p)

GOOD LUCK !