LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T/MTF107**
Examination date Time	2012-09-01 09.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E304
Examiner: Hans Weber Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Wave functions and eigenfunctions**

   Consider a free particle with mass m in one dimension. The wave function of the particle at $t = 0$ is given by
   $$\psi(x, t = 0) = \cos^3(kx) + \sin^5(kx).$$

   (a) Show that the state function $\psi(x, 0)$ can be written as a superposition of eigenfunctions of the free-particle Hamiltonian.

   (b) Determine the energy of each plane wave in the superposition.

   (c) Give the wave function $\psi(x, t)$ at an arbitrary time t. (3 p)

2. **Hydrogen atom**

   Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:
   $$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{15}} \left(2\psi_{100}(\mathbf{r}) - 3\psi_{200}(\mathbf{r}) + \psi_{310}(\mathbf{r}) + \psi_{322}(\mathbf{r})\right)$$

   (a) Is this wave function an eigenfunction of the parity operator $\hat{\Pi}$?

   (b) What is the probability of finding the system in the ground state? In the state (200)? In the state (310)? In the state (322)? In any other state?

   (c) What is the expectation value of the energy; of the operator $\mathbf{L}^2$; of the the operator L_z.

   (3 p)

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3. **Quantum states of Tritium and Helium**

An electron is in the ground state of tritium ^3H. A β-decay instantaneously changes the atom into a helium ion $^3He^+$. The β particle (=high energy electron) leaves the helium ion and is no longer to be taken into consideration. The helium ion that is left behind has one single electron bound to it.

(a) Calculate the probability that the electron (bound to helium ion) is in the 2s-state ($n = 2$, $l = m = 0$) after the decay.

(b) Calculate the probability that the electron is in a 2p-state ($n = 2$, $l = 1$) after the decay.

(c) Calculate the probability that the electron is in a 1s-state ($n = 1$, $l = m = 0$) after the decay.

(d) Is it possible for the electron to receive the quantum numbers ($n = 1$, $l = 1$) after the decay?

(3p)

4. **Parity operator**

The parity operator $\hat{\Pi}$, acts on a function $\Psi(x)$ in the following way $\hat{\Pi}\Psi(x) = \Psi(-x) = \lambda\Psi(x)$ where the equality is valid only if the function Ψ is an eigenfunction to the operator $\hat{\Pi}$, the eigenvalue λ can take two possible values ± 1.

(a) Show that the following functions are eigenfunctions of the parity operator and find the corresponding eigenvalue in each case.

i. $\Psi_1(x) = C\left(\sin(\frac{\pi x}{L}) + \sin(\frac{3\pi x}{L})\right)$

ii. $\Psi_2(x, y, z) = Ce^{-a\sqrt{x^2+y^2+z^2}}$

iii. $\Psi_3(r, \theta, \phi) = Cf(r)\left(\cos(\theta) + cos^3(\theta)\right)\ e^{i\phi}$

(b) If $\psi_+(x, y, z)$ and $\psi_-(x, y, z)$ are eigenfunctions of $\hat{\Pi}$ corresponding to eigenvalues +1 and -1 respectively.

i. Is the function $\Psi = 2\psi_+(x, y, z) + 3\psi_-(x, y, z)$ an eigenfunction of the parity operator?

ii. Show that it is an eigenfunction of $\hat{\Pi}^2$, and find the eigenvalue.

iii. Do the functions e^{-ikx} and e^{ikx} have parity? If yes what are their eigenvalues. If no form linear combinations that have a definite parity and what are their eigenvalues.

(3 p)

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5. **Three–dimensional box well**

A particle is placed in the potential (a 3 dimensional box well)

$$V(x,y,z) = \begin{cases} 0 & \text{for} \quad 0 \le x \le a \text{ and } 0 \le y \le a \text{ and } 0 \le z \le \frac{\mathbf{a}}{\mathbf{2}} \\ +\infty & \text{for} \quad x > a \;\text{ or } x < 0 \;\text{ or } y > a \;\text{ or } y < 0 \;\text{ or } z > \frac{\mathbf{a}}{\mathbf{2}} \;\text{ or } z < 0. \end{cases}$$

(a) Calculate (solve the Schrödinger equation) the eigenfunctions ?

(b) What are the 7 lowest eigenenergies ?

(c) What are the degeneracies of the states associated to these 7 lowest eigenenergies ?

(3p)

GOOD LUCK !