LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date	2013-01-15
Time	15.00 - 20.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber   Tel: (49)2088, Room E304
Examiner: Hans Weber   Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 8.0 points are required to pass the examination. Grades 3: 8.0, 4: 10.0, 5: 12.0

---

1. **Operators and eigenfunctions**

   Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

   (a) $\psi(t) = \cos\omega t$ and $\hat{A} = i\hbar\frac{\partial^2}{\partial t^2}$.

   (b) $\psi(x) = e^{ikx}$ and $\hat{A} = \frac{\partial}{\partial x}$.

   (c) $\psi(x) = e^{-ax^2}$ and $\hat{A} = \frac{\partial}{\partial x}$.

   (d) $\psi(x) = \cos kx$ and $\hat{A} = \frac{\partial}{\partial x}$.

   (e) $\psi(x) = kx$ and $\hat{A} = \frac{\partial}{\partial x}$.

   (f) $\psi(x) = \sin kx$ and $\hat{A} = \hat{P} =$ the parity operator.

   (3 p)

2. **Quantum rotator**

   The Hamiltonian (in units of eV) for a given axially symmetric quantum rotator is

$$H = \frac{L_x^2 + L_y^2}{3\hbar^2} + \frac{L_z^2}{4\hbar^2}$$

   What are the possible energies? (3p)

TURN PAGE!

3. **Time evolution of solution**

A particle of mass m, which moves freely inside a one–dimensional infinite square well potential of length a, has the following initial wave function at time $t = 0$:

$$\psi(x,0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$$

where A is a real constant.

a) Find A so that $\psi(x,0)$ is normalised.

b) If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle $< E >$.

c) Find the wave function $\psi(x,0)$ at any later time t.

(3p)

4. **Expectation values of the Harmonic oscillator in 1d**

For the ground state of the one dimensional Harmonic oscillator calculate:

a) the four expectation values $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$, $\langle p^2 \rangle$ by explicit integration.

b) the uncertainty $\Delta x \cdot \Delta p$ and validate that the uncertainty principle holds.

(3p)

5. **Sudden change in the Harmonic oscillator potential in 1d**

A particle of mass m is in the ground state of the one dimensional Harmonic oscillator potential $V_1(x) = \frac{1}{2} m\omega_1^2 x^2$, when the potential suddenly changes to $V_2(x) = \frac{1}{2} m\omega_2^2 x^2$ without initially changing the wave function.

a) What is the probability that a measurement of the particle energy would yield the result $\frac{1}{2}\hbar\omega_2$?

b) What is the probability that a measurement of the particle energy would yield the result $\frac{3}{2}\hbar\omega_2$?

(3p)

GOOD LUCK !