Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 8.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

1. **Two dimensional Square well**

   A particle is placed in the potential (a 2 dimensional square well)

   $$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ +\infty & \text{for} \quad x > \frac{a}{2}, \ x < -\frac{a}{2} \text{ and } y > \frac{a}{2}, \ y < -\frac{a}{2}. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions !

   (b) Write down the eigenfunctions for the ground state and one for the lowest excited states. Formulate the meaning of orthogonallity and show by explicit calculation that these two eigenfunctions are orthogonal.

   (3p)

2. **Measurement of spin**

   A spin $\frac{1}{2}$ particle is prepared to be in an eigenstate to S_z with eigenvalue $+\frac{1}{2}\hbar$. A subsequent measurement of the spin in the direction $\hat{n} = \sin(\varphi)\hat{e}_y + \cos(\varphi)\hat{e}_z$ is made. The value of φ is $\pi/4$.

   (a) What is the probability to get the value $+\hbar/2$ and $-\hbar/2$ in this new direction $\hat{n}$?

   (b) What would the result (eigenvalue and probability) be of a subsequent measurement in the z-direction of the $+\hbar/2$ state in a) ?

   (3p)

TURN PAGE!

3. Molecular spectra

In the VPL Molecular Spectroscopic Database the two graphs for the Hydrogen Bromide molecular spectra shown in figure 1 are found. The spectra consist of both rotational and vibrational levels.

In the lectures only the vibrational transitions with $\Delta n = \pm 1, 0$ have been covered. In a real material however the potential is not purely harmonic (pure harmonic potential is $V(x) = kx^2$) but other anharmonic (like x^4) contributions exist. Due to these anharmonic contributions also vibrational transitions with higher changes in n are allowed ie $\Delta n = \pm 2$. These transitions are however much rarer as their transition probability is much lower.
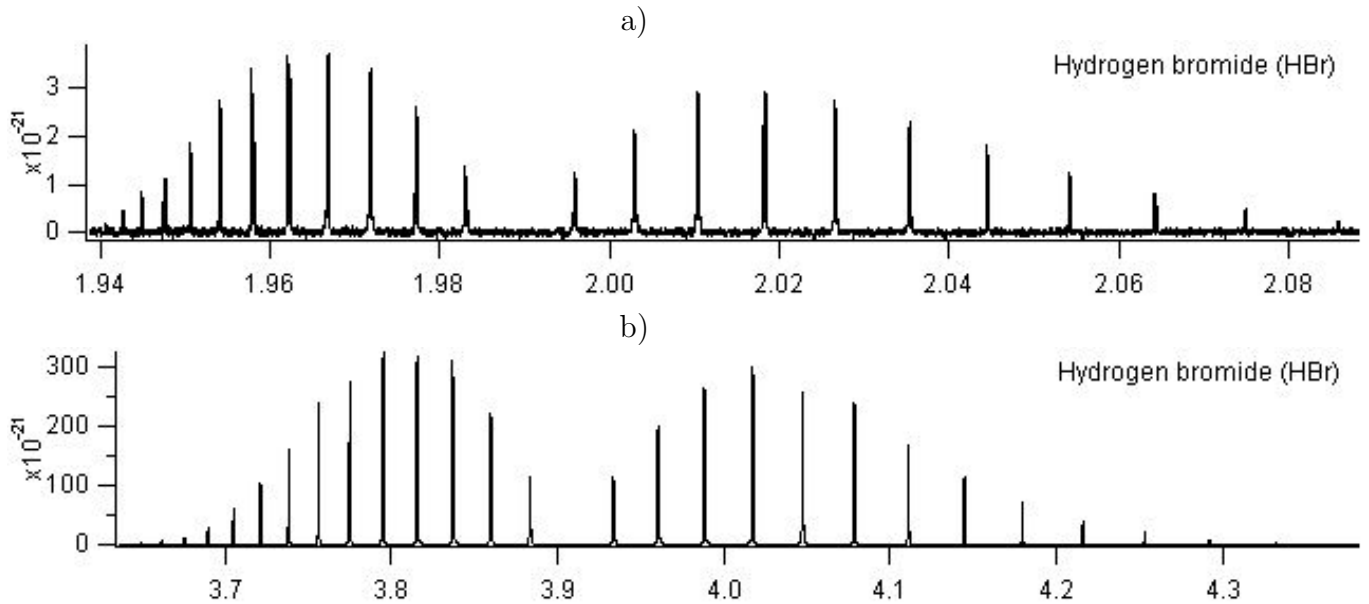
a)

b)

Figure 1: Two spectra for Hydrogen Bromide (HBr) in the micro wave range. Shown is intensity as a function of wavelength. **Note:** the scale on the x–axis is in μm.

(a) Of the two spectra which one (upper/lower) belongs (determine by inspection) to the anharmonic vibrational transition and which one belongs to the harmonic vibrational transition? Motivate !

(b) With data taken from the (the harmonic vibrational spectra) graph evaluate the strength k (in $V(x) = kx^2$) of the coupling constant for the molecule.

(c) Explain the apparent missing line in the middle of the spectra (the harmonic vibrational spectra).

(3p)

4. **Commutator**

   Calculate the commutator $[L_z, sin(\phi)]$, where ϕ is the spherical
   angle $\phi = \arctan(y/x) + n\pi$.   (3 p)

5. **Quantum rotator**

   The Hamiltonian (in units of eV) for a given axially symmetric quantum rotator is (Pay
   attention to the subscripts)

   $$H = \frac{L_z^2 + L_y^2}{2\hbar^2} + \frac{L_x^2}{4\hbar^2}.$$

   Write down an expression for the eigenenergys. How are the quantum numbers related
   ?   (3p)

GOOD LUCK !