LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date	2013-08-31
Time	9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E304
Examiner: Hans Weber Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Compton scattering** A 100-keV photon collides with an electron at rest. The photon is scattered through $\theta = 75^o$. (Note, in the figure 1 below the angle is perhaps not 75^o)

   (a) What is its energy and wavelength of the photon after the collision?

   (b) What is the kinetic energy in eV of the electron after the collision?

   (c) What is the direction of the recoil (electron) ?

(3p)

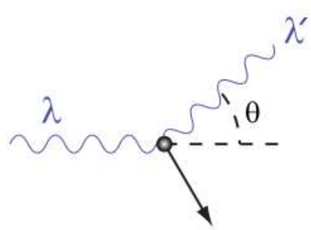

Figure 1: Compton scattering of a photon of wavelength λ through an angle θ to a photon of wavelength λ'.

2. **Spin**

   A spin 1/2 particle described by the unnormalised spinor χ

$$\chi = A \begin{pmatrix} 2+5i \\ 3-i \end{pmatrix}.$$

   (a) Evaluate the expectation values of the three Cartesian components ($< S_x >, < S_y >, < S_z >$).

   (b) For a measurement of spin along the x direction what are the possible outcomes of this experiment and their probabilities?

(3p)

3. **Angular momentum and r in Hydrogen**

   An electron bound in a hydrogen atom is described by the following state:

$$\psi(\boldsymbol{r}) = \psi(x,y,z) = Nxze^{-\sqrt{x^2+y^2+z^2}/3a_0},$$

   where a_0 is the Bohr radius and N is a constant (normalisation).

   (a) A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities.

   (b) Calculate the expectation value of the electrons distance $<r>$ from the nucleus.

(3p)

4. **Two dimensional Square well and parity**

   A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \le x \le \frac{a}{2} \text{ and } -\frac{a}{2} \le y \le \frac{a}{2} \\ \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation !) the eigenfunctions !

   (b) The Hamiltonian commutes with the parity operator P, $P\Psi(x,y) = \Psi(-x,-y) = \lambda\Psi(x,y)$ where the eigenvalue λ can take two possible values ± 1.

   Write down the eigenstates corresponding to the four lowest **energies** in such a way that they are also eigenfunctions of the parity operator P. What is the parity of these states?

(3p)

5. **Perturbation calculation**

   Consider a one–dimensional harmonic oscillator of mass m and angular frequency ω with the Hamiltonian:

$$H^0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2$$

   Due to the surrounding of the oscillator adding an anharmonic term $H^1 = Ax^4$ to the unperturbed H^0 would more correctly describe the system (A is a constant).

   Calculate the energy difference between the first excited level and the ground state of the perturbed system.

   (*Hint*, integrate by parts. For the unperturbed system the energy difference would be $\hbar\omega$)

(3 p)

GOOD LUCK !