LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date	2014-01-14
Time	15.00 - 20.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Nils Almqvist Tel: (49)2291, Room E303
Examiner: Hans Weber Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 8.0 points are required to pass the examination. Grades 3: 8.0, 4: 10.0, 5: 12.0

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1. **Measurement of spin**

   A measurement of the spin component in the direction $\hat{n} = \hat{x}\sin(\varphi) + \hat{y}\cos(\varphi)$ gives the value $-\hbar/2$, where $\hat{x}$ and $\hat{y}$ are unit vectors.

   (a) Calculate the spin state corresponding to this measurement.

   (b) What would the result be of a measurement in the z-direction?

   (c) If one would after the measurement in b) make a new measurement in the direction $\hat{n}$ what would the probability be to get the value $-\hbar/2$ again? Motivate !

   (3p)

2. **Hydrogen atom**

   Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

$$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{15}}\left(3\psi_{100}(\mathbf{r}) - 2\psi_{200}(\mathbf{r}) + \psi_{320}(\mathbf{r}) - \psi_{322}(\mathbf{r})\right)$$

   (a) Is this wave function an eigenfunction of the parity operator $\hat{\Pi}$?

   (b) What is the probability of finding the system in the ground state? In the state (200)? In the state (320)? In the state (322)? In any other state?

   (c) What is the expectation value of the energy (in eV); of the operator $\mathbf{L}^2$ (in units of $\hbar^2$); of the the operator L_z (in units of $\hbar$).

   (3p)

TURN PAGE!

3. **Particle in one dimensional potential**

A particle of mass m moves in one dimension under the influence of a potential $V(x)$. Suppose the particle is in an energy eigenstate $\psi(x) = \left(\frac{\gamma^2}{\pi}\right)^{\frac{1}{4}} e^{-\gamma^2 x^2/2}$ with energy $E = \frac{\hbar^2\gamma^2}{2m}$.

(a) Calculate the expectation value of the position x, ($\langle x \rangle$).

(b) Calculate the expectation value of the momentum p_x, ($\langle p_x \rangle$).

(c) Find $V(x)$.

(3p)

4. **A quantum system at temperature**

A quantum system has **four** eigenstates with energies according to

$$E_{n_1,n_2} = (n_1 + n_2 + 1)\ \hbar\omega$$

where n_1, n_2 are integers $n_i = 0, 1$. The quantum system is coupled to a heatbath of temperature T with which it can exchange energy.

(a) Calculate the partition function of the system for any temperature.

(b) At what temperature T equals the probability to find the quantum system in a state of energy $\hbar\omega$ to find it in a state of energy $2\hbar\omega$?

(c) How large is this probability ?

(3p)

5. **Two dimensional Square well**

A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

(a) Calculate (solve the Schrödinger equation) the eigenfunctions !

(b) Write down the eigenfunctions for the ground state and one for the lowest excited states. Formulate the meaning of orthogonallity and show by explicit calculation that these two eigenfunctions are orthogonal.

(3p)

GOOD LUCK !