LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date Time	2014-03-18 9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber   Tel: (49)2088, Room E304
Examiner: Hans Weber   Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Operators and eigenfunctions**

   Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

   Note: Do not make guesses, a wrong answer will be counted negative ! (No answer to one of the items is counted as zero)

   (a) $\psi(t) = \cos\omega t$ and $\hat{A} = i\hbar\frac{\partial^2}{\partial t^2}$.
   (b) $\psi(x) = e^{ikx}$ and $\hat{A} = \frac{\partial}{\partial x}$.
   (c) $\psi(x) = e^{-ax^2}$ and $\hat{A} = \frac{\partial}{\partial x}$.
   (d) $\psi(x) = \cos kx$ and $\hat{A} = \frac{\partial}{\partial x}$.
   (e) $\psi(x) = kx$ and $\hat{A} = \frac{\partial}{\partial x}$.
   (f) $\psi(x) = \sin kx$ and $\hat{A} = \hat{P}$ = the parity operator.
   (g) $\psi(z) = C(1 + z^2)$ and $\hat{A} = -i\hbar\frac{\partial}{\partial z}$.
   (h) $\psi(z) = Ce^{-3z}$ and $\hat{A} = -i\frac{\hbar}{2}\frac{\partial}{\partial z}$.
   (i) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (0 – 3 p)

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2. **Perturbed Harmonic oscillator**

A particle of mass m is described as a 1 dimensional anharmonic oscillator. The Hamiltonian of the system is

$$H_0 = \frac{p_x^2}{2m} + \frac{m\omega^2}{2}x^2 + \gamma x^4.$$

(a) Assuming that γ is small, use first–order perturbation theory to calculate the ground state energy.

(b) What if the anharmonic perturbation above γx^4 would instead originate from an electric field described by ϵx where ϵ is small. The appropriate Hamiltonian would be:

$$H_0 = \frac{p_x^2}{2m} + \frac{m\omega^2}{2}x^2 + \epsilon x.$$

Use first–order perturbation theory to calculate the energy of the ground state and the lowest excited state.

(3p)

3. **Wave functions and eigenfunctions**

Consider a free particle with mass m in one dimension. The wave function of the particle at $t = 0$ is given by

$$\psi(x, t = 0) = \cos^3(kx) + \sin^3(kx).$$

(a) Show that the state function $\psi(x, 0)$ can be written as a superposition of eigenfunctions of the free-particle Hamiltonian.

(b) Determine the energy of each plane wave in the superposition.

(c) Give the wave function $\psi(x, t)$ at an arbitrary time t. (3 p)

4. **Harmonic oscillator**

A particle of mass m in the harmonic oscillator potential starts out ($t = 0$) in the state

$$\Psi(x, t = 0) = A\left(1 + 2\sqrt{\frac{m\omega}{\hbar}}x\right)^2 e^{-\frac{m\omega}{2\hbar}x^2}$$

for some constant A.

(a) What is the expectation value of the energy ?

(b) At some later time t the wave function is

$$\Psi(x, t) = B\left(1 - 2\sqrt{\frac{m\omega}{\hbar}}x\right)^2 e^{-\frac{m\omega}{2\hbar}x^2}$$

for some constant B (note sign change!). What is the smallest possible value for the time t ?

(3p)

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5. **Eigenfunctions and uncertainty**

An electron confined in a quantum well has five discrete energy levels $E_1 = 0.25$ eV, $E_2 = 0.95$ eV, $E_3 = 2.12$ eV, $E_4 = 3.23$ eV, $E_5 = 4.79$ eV. It is in a state in which the probabilities associated with these energies are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{24}$ and $\frac{1}{24}$ respectively.

(a) Find the expectation value of its energy $\langle \hat{H} \rangle$ and the corresponding uncertainty $\Delta \hat{H}$.

(b) Obtain an expression for the wave function $\Psi(x)$ describing the state of the particle in terms of its energy eigenfunctions $\psi_n(x)$ at time $t = 0$. Why is the expression not unique? Write down two different wave functions corresponding to the same values of $\langle \hat{H} \rangle$ and $\Delta \hat{H}$ that you found in (a).

(c) Assume the potential is the infinite square well of width L, and you would have calculated $\langle \hat{H} \rangle$ in some way. If one adiabatically would increase L to $3L$ by how much would $\langle \hat{H} \rangle$ change? Adiabatically means we are not inducing transitions between levels in the system.

(3 p)

GOOD LUCK !