

Course code	F0047T
Examination date	2014-08-30
Time	9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS

Total number of problems: 5

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

1. Time evolution of a solution

A particle of mass m , which moves freely inside a one-dimensional infinite square well potential of length a , has the following initial wave function at time $t = 0$:

$$\psi(x, 0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$$

where A is a real constant.

- Find A so that $\psi(x, 0)$ is normalised.
- If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle $\langle E \rangle$.
- Find the wave function $\psi(x, 0)$ at any later time t .

(3p)

2. Angular momentum and r in Hydrogen

An electron bound in a hydrogen atom is described by the following state:

$$\psi(\mathbf{r}) = \psi(x, y, z) = Nxze^{-\sqrt{x^2+y^2+z^2}/3a_0},$$

where a_0 is the Bohr radius and N is a constant (normalisation).

- A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities.
- Calculate the expectation value of the electrons distance $\langle r \rangle$ from the nucleus.

(3p)

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3. Molecular spectra

In the VPL Molecular Spectroscopic Database the two graphs for the Hydrogen Bromide molecular spectra shown in figure 1 are found. The spectra consist of both rotational and vibrational levels.

In the lectures only the vibrational transitions with $\Delta n = \pm 1, 0$ have been covered. In a real material however the potential is not purely harmonic (pure harmonic potential is $V(x) = kx^2$) but other anharmonic (like x^4) contributions exist. Due to these anharmonic contributions also vibrational transitions with higher changes in n are allowed i.e. $\Delta n = \pm 2$. These transitions are however much rarer as their transition probability is much lower.

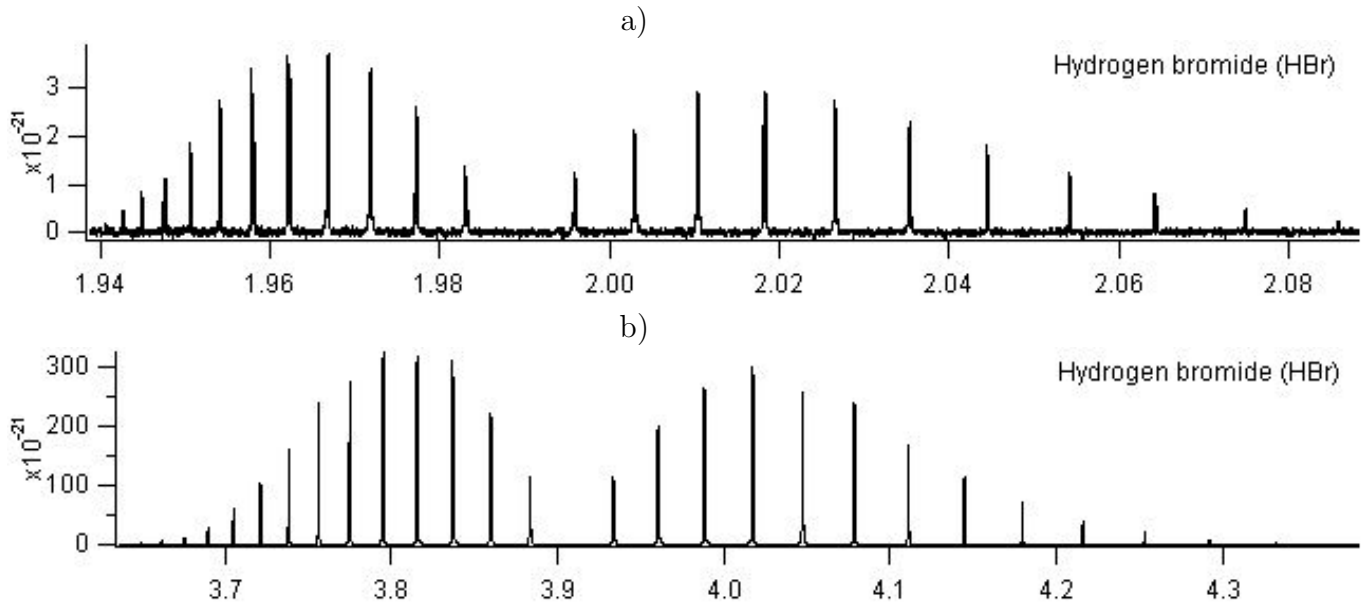


Figure 1: Two spectra for Hydrogen Bromide (HBr) in the micro wave range. Shown is intensity as a function of wavelength. **Note:** the scale on the x-axis is in μm .

- Of the two spectra which one (upper/lower) belongs (determine by inspection) to the anharmonic vibrational transition and which one belongs to the harmonic vibrational transition? Motivate !
- With data taken from the (the harmonic vibrational spectra) graph evaluate the strength k (in $V(x) = kx^2$) of the coupling constant for the molecule.
- Explain the apparent missing line in the middle of the spectra (the harmonic vibrational spectra).

(3p)

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4. Spin

A spin 1/2 particle is described by the unnormalised spinor χ

$$\chi = A \begin{pmatrix} 2 + 5i \\ 3 - i \end{pmatrix}.$$

- (a) Evaluate the expectation values of the three Cartesian components ($\langle S_x \rangle$, $\langle S_y \rangle$, $\langle S_z \rangle$).
- (b) For a measurement of spin along the x direction what are the possible outcomes of this experiment and their probabilities?

(3p)

5. Sudden change in the Harmonic oscillator potential in 1d

A particle of mass m is in the ground state of the one dimensional Harmonic oscillator potential $V_1(x) = \frac{1}{2}m\omega_1^2x^2$, when the potential suddenly changes to $V_2(x) = \frac{1}{2}m\omega_2^2x^2$ without initially changing the wave function.

- a) What is the probability that a measurement of the particle energy would yield the result $\frac{1}{2}\hbar\omega_2$?
- b) What is the probability that a measurement of the particle energy would yield the result $\frac{3}{2}\hbar\omega_2$?

(3p)

GOOD LUCK !