LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date	2015-01-13
Time	15.00 - 20.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E304
Examiner: Hans Weber Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 8.0 points are required to pass the examination. Grades 3: 8.0, 4: 10.0, 5: 12.0

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1. **Time evolution of solution**

   A particle of mass m, which moves freely inside a one–dimensional infinite square well potential of length a, has the following initial wave function at time $t = 0$:

   $$\psi(x,0) = \frac{\sqrt{13}}{\sqrt{8a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{2\sqrt{a}} \sin\left(\frac{5\pi x}{a}\right) + \frac{A}{\sqrt{a}} \sin\left(\frac{7\pi x}{a}\right)$$

   where A is a real constant.

   a) Find A so that $\psi(x,0)$ is normalised.

   b) If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle $< E >$.

   c) Find the wave function $\psi(x,0)$ at any later time t.

   (3p)

2. **Reflection and transmission at a potential step**

   Consider an electron of energy E incident on the potential step $V(x)$,

   $$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

   where $V_0 = 4.5$ eV. Calculate the reflection coefficient R and the transmission coefficient T

   a) when $E = 2.0$ eV,

   b) when $E = 5.0$ eV,

   c) when $E = 7.0$ eV.

   (3p)

TURN PAGE!

3. **Expectation values of the Harmonic oscillator in 1d**

   For the ground state of the one dimensional Harmonic oscillator calculate:

   a) the four expectation values $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$, $\langle p^2 \rangle$ by explicit integration.

   b) the uncertainty $\Delta x \cdot \Delta p$ and validate that the uncertainty principle holds.

(3p)

4. **Parity operator**

   The parity operator $\hat{\Pi}$, acts on a function $\Psi(x)$ in the following way $\hat{\Pi}\Psi(x) = \Psi(-x) = \lambda\Psi(x)$ where the equality is valid only if the function Ψ is an eigenfunction to the operator $\hat{\Pi}$, the eigenvalue λ can take two possible values ± 1.

   (a) Show that the following functions are eigenfunctions of the parity operator and find the corresponding eigenvalue in each case.

      i. $\Psi_1(x) = C\left(\sin(\frac{\pi x}{L}) + \sin(\frac{3\pi x}{L})\right)$

      ii. $\Psi_2(x, y, z) = Ce^{-a\sqrt{x^2+y^2+z^2}}$

      iii. $\Psi_3(r, \theta, \phi) = Cf(r)\left(\cos(\theta) + cos^3(\theta)\right)\ e^{i\phi}$

   (b) If $\psi_+(x, y, z)$ and $\psi_-(x, y, z)$ are eigenfunctions of $\hat{\Pi}$ corresponding to eigenvalues $+1$ and -1 respectively.

      i. Is the function $\Psi = 2\psi_+(x, y, z) + 3\psi_-(x, y, z)$ an eigenfunction of the parity operator?

      ii. Show that it is an eigenfunction of $\hat{\Pi}^2$, and find the eigenvalue.

      iii. Do the functions e^{-ikx} and e^{ikx} have parity? If yes what are their eigenvalues. If no form linear combinations that have a definite parity and what are their eigenvalues.

(3 p)

5. **Two dimensional Square well**

   A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \le x \le \frac{a}{2} \text{ and } -\frac{a}{2} \le y \le \frac{a}{2} \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions !

   (b) Write down the eigenfunctions for the ground state and one for the lowest excited states. Formulate the meaning of orthogonallity and show by explicit calculation that these two eigenfunctions are orthogonal.

(3p)

GOOD LUCK !