## LULEA UNIVERSITY OF TECHNOLOGY

Division of Physics

| Course code | F0047T |
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| Examination date | $2015-03-17 /$ E241 |
| Time | $9.00-14.00$ (5 hours) |

Examination in: Kvantfysik / Quantum Physics
Total number of problems: 5
Teacher on duty: Nils Almqvist Tel: (49)2291, Room E303
Examiner: Hans Weber
Tel: (49)2088, Room E304
Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, Collection of formulae
Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p .7 .0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

## 1. Angular momentum

Suppose an electron is in a state described by the wavefunction

$$
\psi=\frac{1}{\sqrt{4 \pi}}\left(e^{i \phi} \sin (\theta)+\cos (\theta)\right) g(r)
$$

where

$$
\int_{0}^{\infty}|g(r)|^{2} r^{2} d r=1
$$

and $\phi, \theta$ are the azimuth and polar angles respectively.
(a) What are the possible results of a measurement of the z-component $L_{z}$ of the angular momentum of the electron in this state?
(b) What is the probability of obtaining each of the possible results in part (a)?
(c) What are the expectation values of $L_{z}$ and $L^{2}$ ?

## 2. Molecular spectra

In the rotational fine structure spectra of ${ }^{1} \mathrm{H}^{35} \mathrm{Cl}$ the following spectral lines where detected 2824,0 $\mathrm{cm}^{-1} ; 2844,6 \mathrm{~cm}^{-1} ; 2865,3 \mathrm{~cm}^{-1} ; 2906,7 \mathrm{~cm}^{-1} ; 2927,4 \mathrm{~cm}^{-1} ; 2948,0$ $\mathrm{cm}^{-1}$ och $2968,7 \mathrm{~cm}^{-1}$.
a) There seems to be a line missing, explain why.
b) Using the data given calculate the distance between the atoms of the molecule.

## 3. Quantum states of Tritium and Helium

An electron is in the ground state of tritium ${ }^{3} \mathrm{H}$. A $\beta$-decay instantaneously changes the atom into a helium ion ${ }^{3} \mathrm{He}^{+}$. The $\beta$ particle (=high energy electron) leaves the helium ion and is no longer to be taken into consideration. The helium ion that is left behind has one single electron bound to it.
(a) Calculate the probability that the electron (bound to helium ion) is in the 2 s-state ( $n=2, l=m=0$ ) after the decay.
(b) Calculate the probability that the electron is in a 2 p-state $(n=2, l=1)$ after the decay.
(c) Calculate the probability that the electron is in a 1 s-state $(n=1, l=m=0)$ after the decay.
(d) Is it possible for the electron to receive the quantum numbers $(n=1, l=1)$ after the decay?

## 4. Compton scattering

A $100-\mathrm{keV}$ photon collides with an electron at rest. The photon is scattered through $\theta=75^{\circ}$. (Note, in the figure 1 below the angle is perhaps not $75^{\circ}$ )
(a) What is its energy and wavelength of the photon after the collision?
(b) What is the kinetic energy in eV of the electron after the collision?
(c) What is the direction of the recoil (electron)?


Figure 1: Compton scattering of a photon of wavelength $\lambda$ through an angle $\theta$ to a photon of wavelength $\lambda^{\prime}$.

## 5. Measurement of spin

A measurement of the spin component in the direction $\hat{n}=\hat{x} \sin (\varphi)+\hat{y} \cos (\varphi)$ gives the value $+\hbar / 2$, where $\hat{x}$ and $\hat{y}$ are unit vectors.
(a) Calculate the spin state corresponding to this measurement.
(b) What would the result be of a measurement in the $z$-direction?
(c) If one would after the measurement in b) make a new measurement in the direction $\hat{n}$ what would the probability be to get the value $+\hbar / 2$ again? Motivate !

