LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date	2015-08-29
Time	9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E304
Examiner: Hans Weber Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Time evolution of a solution**

   A particle of mass m, which moves freely inside a one–dimensional infinite square well potential of length a, has the following initial wave function at time $t = 0$:

$$\psi(x,0) = \sqrt{\frac{11}{8a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{2\sqrt{a}} \sin\left(\frac{4\pi x}{a}\right) + \frac{A}{\sqrt{a}} \sin\left(\frac{5\pi x}{a}\right)$$

   where A is a real constant.

   a) Find A so that $\psi(x,0)$ is normalised.

   b) If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle $< E >$.

   c) Find the wave function $\psi(x,t)$ at any later time t.

   (3p)

2. **Quantum rotator**

   The Hamiltonian (in units of eV) for a given axially symmetric quantum rotator is

$$H = \frac{L_x^2 + L_y^2}{2\hbar^2} + \frac{L_z^2}{3\hbar^2}$$

   What are the possible energies? (3p)

TURN PAGE!

3. **Measurement of spin**

   A spin $\frac{1}{2}$ particle is prepared to be in an eigenstate to S_z with eigenvalue $+\frac{1}{2}\hbar$. A subsequent measurement of the spin in the direction $\hat{n} = \sin(\varphi)\hat{e}_y + \cos(\varphi)\hat{e}_z$ is made. The value of φ is $\pi/4$.

   (a) What is the probability to get the value $+\hbar/2$ and $-\hbar/2$ in this new direction $\hat{n}$?

   (b) What would the result (eigenvalue and probability) be of a subsequent measurement in the z-direction of the $+\hbar/2$ state in a) ?

   (3p)

4. **Three–dimensional box well**

   A particle is placed in the potential (a 3 dimensional box well)

$$V(x,y,z) = \begin{cases} 0 & \text{for} \quad 0 \le x \le a \text{ and } 0 \le y \le a \text{ and } 0 \le z \le \frac{\mathbf{a}}{\mathbf{2}} \\ +\infty & \text{for} \quad x > a \ \text{ or } x < 0 \ \text{ or } y > a \ \text{ or } y < 0 \ \text{ or } z > \frac{\mathbf{a}}{\mathbf{2}} \ \text{ or } z < 0. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions ?

   (b) What are the 7 lowest eigenenergies ?

   (c) What are the degeneracies of the states associated to these 7 lowest eigenenergies ?

   (3p)

5. **Perturbed Harmonic oscillator**

   A particle of mass m is described as a 1 dimensional anharmonic oscillator. The Hamiltonian of the system is

$$H_0 = \frac{p_x^2}{2m} + \frac{m\omega^2}{2}x^2 + \gamma x^4.$$

   (a) Assuming that γ is small, use first–order perturbation theory to calculate the ground state energy.

   (b) What if the anharmonic perturbation above γx^4 would instead originate from an electric field described by ϵx where ϵ is small. The appropriate Hamiltonian would be:

$$H_0 = \frac{p_x^2}{2m} + \frac{m\omega^2}{2}x^2 + \epsilon x.$$

   Use first–order perturbation theory to calculate the energy of the ground state and the lowest excited state.

   (3p)

GOOD LUCK !