

Course code	F0047T
Examination date	2016-01-12
Time	15.00 - 20.00 (5 hours)

Examination in: **KVANTFYSIK / QUANTUM PHYSICS**

Total number of problems: 5

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 8.0 points are required to pass the examination. Grades 3: 8.0, 4: 10.0, 5: 12.0

1. Angular momentum and r in Hydrogen

An electron bound in a hydrogen atom is described by the following state:

$$\psi(\mathbf{r}) = \psi(x, y, z) = Nxye^{-\sqrt{x^2+y^2+z^2}/3a_0},$$

where a_0 is the Bohr radius and N is a constant (normalisation).

- A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities.
- Calculate the expectation value of the electrons distance $\langle r \rangle$ from the nucleus.

(3p)

2. A quantum system at temperature

A quantum system has **four** eigenstates with energies according to

$$E_{n_1, n_2} = (n_1 + n_2 + 1) \hbar\omega$$

where n_1, n_2 are integers $n_i = 0, 1$. The quantum system is coupled to a heatbath of temperature T with which it can exchange energy.

- Calculate the partition function of the system for any temperature.
- At what temperature T equals the probability to find the quantum system in a state of energy $\hbar\omega$ to find it in a state of energy $2\hbar\omega$?
- How large is this probability ?

(3p)

3. Molecular spectra

Carbon monoxide is the second-most common molecule in the interstellar medium, after molecular hydrogen. Interstellar CO was first detected with radio telescopes in 1970.

In the VPL Molecular Spectroscopic Database the two graphs for the Carbon monoxide molecular spectra shown in figure 1 are found. The spectra consist of both rotational and vibrational levels.

In the lectures only the vibrational transitions with $\Delta n = \pm 1, 0$ have been covered. In a real material however the potential is not purely harmonic (pure harmonic potential is $V(x) = kx^2$) but other anharmonic (like x^4) contributions exist. Due to these anharmonic contributions also vibrational transitions with higher changes in n are allowed i.e. $\Delta n = \pm 2$. These transitions are however much rarer as their transition probability is much lower.

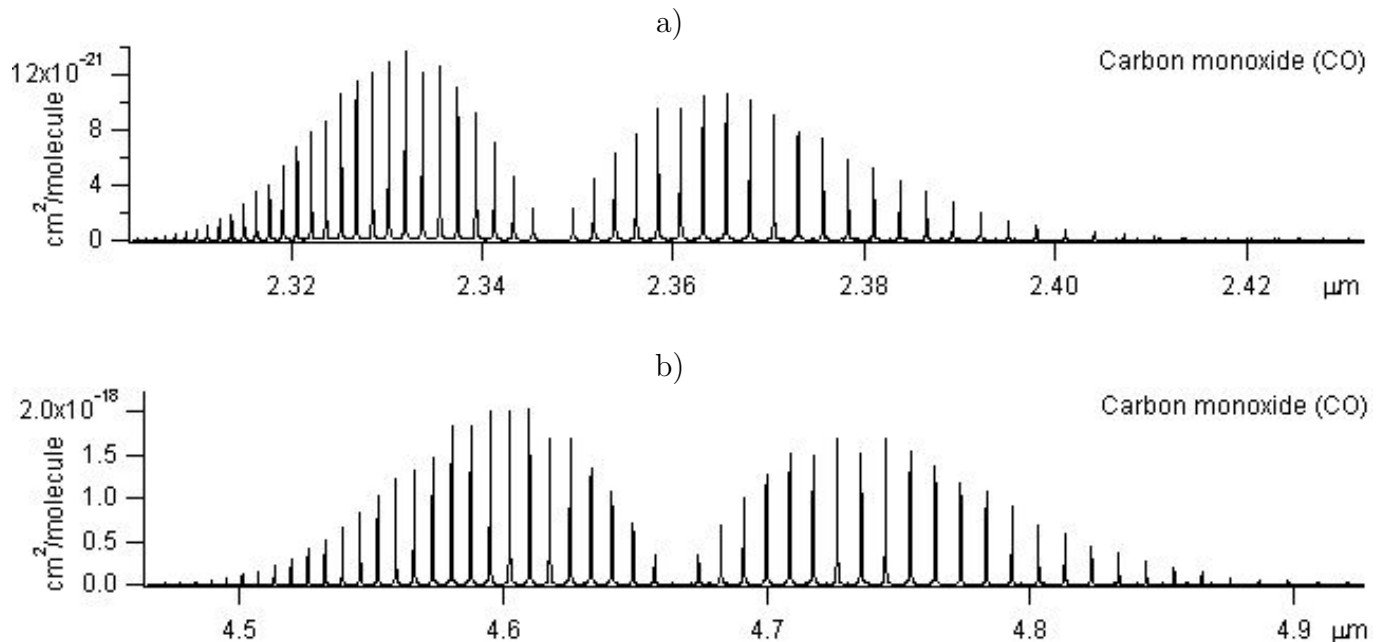


Figure 1: Two spectra for Carbon monoxide (CO) in the micro wave range. Shown is intensity as a function of wavelength. **Note:** the scale on the x-axis is in μm .

- Of the two spectra which one (upper/lower) belongs (determine by inspection) to the anharmonic vibrational transition and which one belongs to the harmonic vibrational transition? Motivate !
- With data taken from the (the harmonic vibrational spectra) graph calculate the distance between the two atoms of the molecule.
- Explain the apparent missing line in the middle of the spectra (the harmonic vibrational spectra).

(3p)

TURN PAGE!

4. Spin

Evaluate for a spin 1/2 particle described by the spinor χ the expectation values of the 3 cartesian components ($\langle S_x \rangle$, $\langle S_y \rangle$, $\langle S_z \rangle$) of the spin and also their squares ($\langle S_x^2 \rangle$, $\langle S_y^2 \rangle$, $\langle S_z^2 \rangle$)

$$\chi = \frac{1}{3} \begin{pmatrix} 2 - i \\ 2 \end{pmatrix}.$$

(3p)

5. Operators and eigenfunctions

Are the following functions ψ eigenfunctions of the given operators \hat{A} ?

Note: Do not make guesses, a wrong answer will be counted negative ! (No answer to one of the items is counted as zero)

- (a) $\psi(t) = \cos \omega t$ and $\hat{A} = i\hbar \frac{\partial^2}{\partial t^2}$.
- (b) $\psi(x) = e^{ikx}$ and $\hat{A} = \frac{\partial}{\partial x}$.
- (c) $\psi(x) = e^{-ax^2}$ and $\hat{A} = \frac{\partial}{\partial x}$.
- (d) $\psi(x) = \cos kx$ and $\hat{A} = \frac{\partial}{\partial x}$.
- (e) $\psi(x) = kx$ and $\hat{A} = \frac{\partial}{\partial x}$.
- (f) $\psi(x) = \sin kx$ and $\hat{A} = \hat{P}$ = the parity operator.
- (g) $\psi(z) = C(1 + z^2)$ and $\hat{A} = -i\hbar \frac{\partial}{\partial z}$.
- (h) $\psi(z) = Ce^{-3z}$ and $\hat{A} = -i\frac{\hbar}{2} \frac{\partial}{\partial z}$.
- (i) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

(0 - 3 p)

GOOD LUCK !