LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date	2016-03-15
Time	9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E304
Examiner: Hans Weber Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Reflection and transmission at a potential step**

   Consider an electron of energy E incident on the potential step $V(x)$,

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

   where $V_0 = 4.5$ eV. Calculate the reflection coefficient R and the transmission coefficient T

   a) when $E = 2.0$ eV,
   b) when $E = 5.0$ eV,
   c) when $E = 7.0$ eV.

(3p)

2. **Two dimensional Square well**

   A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions !
   (b) Write down the eigenfunctions for the ground state and one for the lowest excited states. Formulate the meaning of orthogonallity and show by explicit calculation that these two eigenfunctions are orthogonal.

(3p)

TURN PAGE!

3. **Wave functions and eigenfunctions**

Consider a free particle with mass m in one dimension. The wave function of the particle at $t = 0$ is given by

$$\psi(x, t = 0) = \cos^3(kx) + \sin^3(kx).$$

(a) Show that the state function $\psi(x, 0)$ can be written as a superposition of eigenfunctions of the free-particle Hamiltonian.

(b) Determine the energy of each plane wave in the superposition.

(c) Give the wave function $\psi(x, t)$ at an arbitrary time t. (3 p)

4. **Measurement of spin**

A spin $\frac{1}{2}$ particle is prepared to be in an eigenstate to S_z with eigenvalue $+\frac{1}{2}\hbar$. A subsequent measurement of the spin in the direction $\hat{n} = \sin(\varphi)\hat{e}_y + \cos(\varphi)\hat{e}_z$ is made. The value of φ is $\pi/4$.

(a) What is the probability to get the value $+\hbar/2$ and $-\hbar/2$ in this new direction $\hat{n}$?

(b) What would the result (eigenvalue and probability) be of a subsequent measurement in the z-direction of the $+\hbar/2$ state in a) ?

(3p)

5. **Misc.**

a) Evaluate the commutator $[y^2, p_y^2]$.

b) The ion Be^{3+} has the nuclear charge +4 but only one electron. How much energy does it take to excite the electron from the ground state to the level 2s? Give a numerical value in electron Volts (eV)!

c) The wave function of a hydrogen atom in an eigenstate to the Hamilton operator is:

$$\Psi(r, \theta, \phi) = \frac{1}{81\sqrt{6\pi}}(1/a_\mu)^{3/2}(r^2/a_\mu^2)e^{-r/3a_\mu}[3\cos^2\theta - 1],$$

where a_μ is the Bohr radius (with the reduced mass). Determine the quantum numbers n, l och m_l.

(3p)

GOOD LUCK !