LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0047T**
Examination date Time	2016-08-27 9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber   Tel: (49)2088, Room E163
Examiner: Hans Weber   Tel: (49)2088, Room E163

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Two dimensional Square well**

   A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 \le x \le a \text{ and } 0 \le y \le \frac{a}{2} \\ +\infty & \text{for} \quad x > a,\ x < 0 \text{ and } y > \frac{a}{2},\ y < 0. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions $\psi_{n,m}(x,y)$!

   (b) Write down the eigenfunctions for the ground state $\psi_{1,1}(x,y)$ and one for the lowest excited states ($\psi_{2,1}(x,y)$ or $\psi_{1,2}(x,y)$). Formulate the meaning of orthogonallity and show by explicit calculation that these two eigenfunctions are orthogonal.

(3p)

2. **Hydrogen atom**

   Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

$$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{15}} \left(3\psi_{100}(\mathbf{r}) - 2\psi_{200}(\mathbf{r}) + \psi_{320}(\mathbf{r}) - \psi_{322}(\mathbf{r})\right)$$

   (a) Is this wave function an eigenfunction of the parity operator $\hat{\Pi}$?

   (b) What is the probability of finding the system in the ground state? In the state (200)? In the state (320)? In the state (322)? In any other state?

   (c) What is the expectation value of the energy (in eV); of the operator $\mathbf{L}^2$ (in units of $\hbar^2$); of the the operator L_z (in units of $\hbar$).

(3p)

TURN PAGE!

3. **Perturbation calculation**

Consider a one–dimensional harmonic oscillator of mass m and angular frequency ω with the Hamiltonian:

$$H^0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2$$

Due to the surrounding of the oscillator adding an anharmonic term $H^1 = Ax^4$ to the unperturbed H^0 would more correctly describe the system (A is a constant).

Calculate the energy difference between the first excited level and the ground state of the perturbed system.

(*Hint*, integrate by parts. For the unperturbed system the energy difference would be $\hbar\omega$)

(3p)

4. **Particle in one dimensional potential**

A particle of mass m moves in one dimension under the influence of a potential $V(x)$. Suppose the particle is in an energy eigenstate $\psi(x) = \left(\frac{\gamma^2}{\pi}\right)^{\frac{1}{4}} e^{-\gamma^2 x^2/2}$ with energy $E = \frac{\hbar^2\gamma^2}{2m}$.

   (a) Calculate the expectation value of the position x, ($\langle x \rangle$).

   (b) Calculate the expectation value of the momentum p_x, ($\langle p_x \rangle$).

   (c) Find $V(x)$.

(3p)

5. **A quantum system at temperature**

A quantum system has **four** eigenstates with energies according to

$$E_{n_1,n_2} = (n_1 + n_2 + 1)\,\hbar\omega$$

where n_1, n_2 are integers $n_i = 0, 1$. The quantum system is coupled to a heatbath of temperature T with which it can exchange energy.

   (a) Calculate the partition function of the system for any temperature.

   (b) At what temperature T equals the probability to find the quantum system in a state of energy $\hbar\omega$ to find it in a state of energy $2\hbar\omega$?

   (c) How large is this probability ?

(3p)

GOOD LUCK !