LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date	2017-01-10
Time	15.00 - 20.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysika, Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of points is 15 p. 8.5 points are required to pass the examination. Grades 3: 8.5, 4: 10.5, 5: 12.0 . This includes the bonus points from the three home assignements.

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1. **Two dimensional Square well**

   A particle is placed in the potential (a 2 dimensional square well)

$$V(x,y) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions !

   (b) Write down the eigenfunctions for the ground state and one for the lowest excited states. Formulate the meaning of orthogonallity and show by explicit calculation that these two eigenfunctions are orthogonal.

   (3p)

2. **Time evolution of a harmonic oscillator**

   A one-dimensional harmonic oscillator is at time $t = 0$ described by the wave function $\Psi(x, t = 0) = \frac{1}{\sqrt{2}}(\psi_0 + \psi_1)$, where ψ_n are the energy eigenstates with energy eigenvalues $E_n = \hbar\omega(n + \frac{1}{2})$.

   (a) Give the state $\Psi(x,t)$ at a later time t.

   (b) For this state, determine the expectation value of the kinetic energy $E_k = \frac{p_{op}^2}{2m}$ as a function of time.

   (3p)

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3. **Parity operator**

The parity operator $\hat{\Pi}$, acts on a function $\Psi(x)$ in the following way $\hat{\Pi}\Psi(x) = \Psi(-x) = \lambda\Psi(x)$ where the equality is valid only if the function Ψ is an eigenfunction to the operator $\hat{\Pi}$, the eigenvalue λ can take two possible values ± 1.

(a) Show that the following functions are eigenfunctions of the parity operator and find the corresponding eigenvalue in each case.

i. $\Psi_1(x) = C\left(\cos(\frac{\pi x}{L}) + \cos(\frac{3\pi x}{L})\right)$

ii. $\Psi_2(x, y, z) = Ce^{-a\sqrt{x^2+3y^2+z^2}}$

iii. $\Psi_3(r, \theta, \phi) = Cf(r)\left(\cos(\theta) + cos^3(\theta)\right)\ e^{i\phi}$

(b) If $\psi_+(x, y, z)$ and $\psi_-(x, y, z)$ are eigenfunctions of $\hat{\Pi}$ corresponding to eigenvalues +1 and -1 respectively.

i. Is the function $\Psi = 4\psi_+(x, y, z) + 2\psi_-(x, y, z)$ an eigenfunction of the parity operator?

ii. Show that it is an eigenfunction of $\hat{\Pi}^2$, and find the eigenvalue.

iii. Do the functions $e^{-\alpha x}$ and $e^{\alpha x}$ have parity? If yes what are their eigenvalues. If no form possible linear combinations that have a definite parity and what are their eigenvalues.

(3 p)

4. **Perturbation calculation**

A particle of mass $m = m_e$ is in a infinite square well potential of width $a = 10.0$Å. Due to imperfections the potential takes the following form:

$$V(x) = \begin{cases} \infty & \text{for} \quad x < 0, \\ \epsilon = 0.093\text{eV} & \text{for} \quad 0 < x < \frac{a}{2}, \\ 0 & \text{for} \quad \frac{a}{2} < x < a, \\ \infty & \text{for} \quad x > a \end{cases}$$

(a) Use perturbation theory to calculate to lowest order the corrections to the energy of the ground state and the first excited state. Give both the unperturbed levels and the perturbed ones (in eV).

(b) Now if $\epsilon = 1.09$ eV, would the perturbation calculation yield a good value for the perturbed levels? Motivate your answer!

(3 p)

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5. **Hydrogen atom**

Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

$$\Psi(\mathbf{r}, t = 0) = A\left(2\psi_{100}(\mathbf{r}) - 3\psi_{211}(\mathbf{r}) + \psi_{320}(\mathbf{r}) - \psi_{322}(\mathbf{r})\right)$$

(a) Determine A so that the wave function is normalised.

(b) What is the probability of finding the system in the ground state? In the state (211)? In the state (320)? In the state (322)? In any other state?

(c) What is the expectation value of the energy (in eV); of the operator $\mathbf{L}^2$ (in units of $\hbar^2$); of the the operator L_z (in units of $\hbar$).

(3p)

GOOD LUCK !