LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0047T**
Examination date	2017-03-14
Time	9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Nils Almqvist Tel: (49)2291, Room E303
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 10.0, 5: 12.0

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1. **Two dimensional Square well and parity**

   A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation !) the eigenfunctions !

   (b) The Hamiltonian commutes with the parity operator P, $P\Psi(x,y) = \Psi(-x,-y) = \lambda\Psi(x,y)$ where the eigenvalue λ can take two possible values ± 1.
   Write down the eigenstates corresponding to the four lowest **energies** in such a way that they are also eigenfunctions of the parity operator P. What is the parity of these states?

(3p)

2. **Misc.**

   a) Evaluate the commutator $[x^2, p_x^2]$.

   b) Li^{2+} has the nuclear charge +3 but only one electron. How much energy does it take to excite the electron from the ground state to the level 2p? Give a numerical value in electron Volts (eV)!

   c) The wave function of a hydrogen atom in an eigenstate to the Hamilton operator is:

$$\Psi(r,\theta,\phi) = \frac{1}{81\sqrt{6\pi}}(1/a_\mu)^{3/2}(r^2/a_\mu^2)e^{-r/3a_\mu}[3\cos^2\theta - 1],$$

   where a_μ is the Bohr radius (with the reduced mass). Determine the quantum numbers n, l och m_l.

(3p)

TURN PAGE!

3. **Quantum rotator**

The Hamiltonian (in units of eV) for a given axially symmetric quantum rotator is (Pay attention to the subscripts)

$$H = \frac{L_z^2 + L_y^2}{2\hbar^2} + \frac{L_x^2}{4\hbar^2}.$$

Write down an expression for the eigenenergys. How are the quantum numbers related ? (3p)

4. **Time evolution of a solution**

A particle of mass m, which moves freely inside a one–dimensional infinite square well potential of length a, has the following initial wave function at time $t = 0$:

$$\psi(x, 0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$$

where A is a real constant.

a) Find A so that $\psi(x, 0)$ is normalised.

b) If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle $< E >$.

c) Find the wave function $\psi(x, 0)$ at any later time t.

(3p)

5. **Particle in one dimensional potential**

A particle of mass m moves in one dimension under the influence of a potential $V(x)$. Suppose the particle is in an energy eigenstate $\psi(x) = \left(\frac{\gamma^2}{\pi}\right)^{\frac{1}{4}} e^{-\gamma^2 x^2/2}$ with energy $E = \frac{\hbar^2\gamma^2}{2m}$.

(a) Calculate the expectation value of the position x, ($\langle x \rangle$).

(b) Calculate the expectation value of the momentum p_x, ($\langle p_x \rangle$).

(c) Find $V(x)$.

(3p)

GOOD LUCK !