LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0047T**
Examination date Time	2017-08-26 9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 10.0, 5: 12.0

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1. **Reflection and transmission at a potential step**

   Consider an electron of energy E incident on the potential step $V(x)$,

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

   where $V_0 = 4.5$ eV. Calculate the reflection coefficient R and the transmission coefficient T

   a) when $E = 2.0$ eV,
   b) when $E = 5.0$ eV,
   c) when $E = 7.0$ eV.

(3p)

2. **Measurement of spin**

   A spin $\frac{1}{2}$ particle is prepared to be in an eigenstate to S_z with eigenvalue $+\frac{1}{2}\hbar$. A subsequent measurement of the spin in the direction $\hat{n} = \sin(\varphi)\hat{e}_y + \cos(\varphi)\hat{e}_z$ is made. The value of φ is $\pi/4$.

   (a) What is the probability to get the value $+\hbar/2$ and $-\hbar/2$ in this new direction $\hat{n}$?
   (b) What would the result (eigenvalue and probability) be of a subsequent measurement in the z-direction of the $+\hbar/2$ state in a) ?

(3p)

TURN PAGE!

3. **Wave functions and eigenfunctions**

Consider a free particle with mass m in one dimension. The wave function of the particle at $t = 0$ is given by

$$\psi(x, t = 0) = \cos^3(kx) + \sin^3(kx).$$

(a) Show that the state function $\psi(x, 0)$ can be written as a superposition of eigenfunctions of the free-particle Hamiltonian.

(b) Determine the energy of each plane wave in the superposition.

(c) Give the wave function $\psi(x, t)$ at an arbitrary time t. (3 p)

4. **Perturbed Harmonic oscillator**

A particle of mass m is described as a 1 dimensional anharmonic oscillator. The Hamiltonian of the system is

$$H = \frac{p_x^2}{2m} + \frac{m\omega^2}{2}x^2 + \gamma x^4.$$

(a) Assuming that γ is small, use first–order perturbation theory to calculate the ground state energy.

(b) What if the anharmonic perturbation above γx^4 would instead originate from an electric field described by ϵx where ϵ is small. The appropriate Hamiltonian would be:

$$H = \frac{p_x^2}{2m} + \frac{m\omega^2}{2}x^2 + \epsilon x.$$

Use first–order perturbation theory to calculate the energy of the ground state and the lowest excited state.

(3p)

5. **Angular momentum**

Suppose an electron is in a state described by the wavefunction

$$\psi = \frac{1}{\sqrt{4\pi}}\left(e^{i\phi}\sin(\theta) + \cos(\theta)\right)g(r),$$

where

$$\int_0^\infty |\, g(r)\, |^2\, r^2 dr = 1,$$

and ϕ, θ are the azimuth and polar angles respectively.

(a) What are the possible results of a measurement of the z-component L_z of the angular momentum of the electron in this state?

(b) What is the probability of obtaining each of the possible results in part (a) ?

(c) What are the expectation values of L_z and L^2 ?

(3p)

GOOD LUCK !