LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date Time	2018-01-09 9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysika, Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of points is 15 p. 8.5 points are required to pass the examination. Grades 3: 8.5, 4: 10.5, 5: 12.0 . This includes the bonus points from the three home assignements.

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1. **Operators and eigenfunctions**

   Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

   Note: Do not make guesses, a wrong answer will be counted negative ! (No answer to one of the items is counted as zero)

   (a) $\psi(t) = \cos\omega t$ and $\hat{A} = i\hbar\frac{\partial^2}{\partial t^2}$.
   (b) $\psi(x) = e^{ikx}$ and $\hat{A} = \frac{\partial}{\partial x}$.
   (c) $\psi(x) = e^{-ax^2}$ and $\hat{A} = \frac{\partial}{\partial x}$.
   (d) $\psi(x) = \cos kx$ and $\hat{A} = \frac{\partial}{\partial x}$.
   (e) $\psi(x) = kx$ and $\hat{A} = \frac{\partial}{\partial x}$.
   (f) $\psi(x) = \sin kx$ and $\hat{A} = \hat{P}$ = the parity operator.
   (g) $\psi(z) = C(1+z^2)$ and $\hat{A} = -i\hbar\frac{\partial}{\partial z}$.
   (h) $\psi(z) = Ce^{-3z}$ and $\hat{A} = -i\frac{\hbar}{2}\frac{\partial}{\partial z}$.
   (i) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (0 – 3 p)

2. **Spin**

   A spin 1/2 particle is described by the unnormalised spinor χ

   $$\chi = A\begin{pmatrix} 2+5i \\ 3-i \end{pmatrix}.$$

   (a) Evaluate the expectation values of the three Cartesian components ($< S_x >, < S_y >, < S_z >$).
   (b) For a measurement of spin along the x direction what are the possible outcomes of this experiment and their probabilities?

   (3p)

TURN PAGE!

3. **Angular momentum and r in Hydrogen**

An electron bound in a hydrogen atom is described by the following state:

$$\psi(\boldsymbol{r}) = \psi(x, y, z) = Nxye^{-\sqrt{x^2+y^2+z^2}/3a_0},$$

where a_0 is the Bohr radius and N is a constant (normalisation).

(a) A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities.

(b) Calculate the expectation value of the electrons distance $< r >$ from the nucleus.

(3p)

4. **Eigenfunctions and uncertainty**

An electron confined in a quantum well has five discrete energy levels $E_1 = 0.25$ eV, $E_2 = 0.95$ eV, $E_3 = 2.12$ eV, $E_4 = 3.23$ eV, $E_5 = 4.79$ eV. It is in a state in which the probabilities associated with these energies are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{24}$ and $\frac{1}{24}$ respectively.

(a) Find the expectation value of its energy $\langle \hat{H} \rangle$ and the corresponding uncertainty $\Delta \hat{H}$.

(b) Obtain an expression for the wave function $\Psi(x)$ describing the state of the particle in terms of its energy eigenfunctions $\psi_n(x)$ at time $t = 0$. Why is the expression not unique? Write down two different wave functions corresponding to the same values of $\langle \hat{H} \rangle$ and $\Delta \hat{H}$ that you found in (a).

(c) Assume the potential is the infinite square well of width L, and you would have calculated $\langle \hat{H} \rangle$ in some way. If one adiabatically would increase L to $3L$ by how much would $\langle \hat{H} \rangle$ change? Adiabatically means we are not inducing transitions between levels in the system.

(3 p)

5. **Quantum states of Tritium and Helium**

An electron is in the ground state of tritium ^3H. A β-decay instantaneously changes the atom into a helium ion $^3He^+$. The β particle (=high energy electron) leaves the helium ion and is no longer to be taken into consideration. The helium ion that is left behind has one single electron bound to it.

(a) Calculate the probability that the electron (bound to helium ion) is in the 2s-state ($n = 2$, $l = m = 0$) after the decay.

(b) Calculate the probability that the electron is in a 2p-state ($n = 2$, $l = 1$) after the decay.

(c) Calculate the probability that the electron is in a 1s-state ($n = 1$, $l = m = 0$) after the decay.

(d) Is it possible for the electron to receive the quantum numbers ($n = 1$, $l = 1$) after the decay?

(3p)

GOOD LUCK !