LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0047T**
Examination date	2018-03-13
Time	9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 10.0, 5: 12.0

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1. **Operators and eigenfunctions**

   Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

   Note: Do not make guesses, a wrong answer will be counted negative ! (No answer to one of the items is counted as zero)

   (a) $\psi(t) = \sin(\omega t)\cos(\omega t)$ and $\hat{A} = i\hbar\frac{\partial^2}{\partial t^2}$.

   (b) $\psi(t) = \cos^2(\omega t) - \sin^2(\omega t)$ and $\hat{A} = i\hbar\frac{\partial^2}{\partial t^2}$.

   (c) $\psi(x) = \sin kx$ and $\hat{A} = \frac{\partial}{\partial x}$.

   (d) $\psi(x) = kx^2$ and $\hat{A} = \frac{\partial}{\partial x}$.

   (e) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (f) $\psi(x) = e^{ikx} + e^{-ikx}$ and $\hat{A} = \frac{\partial}{\partial x}$.

   (g) $\psi(x) = \cos kx$ and $\hat{A} = \hat{P} =$ the parity operator.

   (h) $\psi(z) = Ce^{-\omega z}$ and $\hat{A} = -i\frac{\hbar}{2}\frac{\partial}{\partial z}$.

   (i) $\psi(z) = C(1 + z^3)$ and $\hat{A} = -i\hbar\frac{\partial}{\partial z}$.

   (0 – 3 p)

2. **Time evolution of a harmonic oscillator**

   A one-dimensional harmonic oscillator is at time $t = 0$ described by the wave function $\Psi(x, t = 0) = \frac{1}{\sqrt{2}}(\psi_0 + \psi_1)$, where ψ_n are the energy eigenstates with energy eigenvalues $E_n = \hbar\omega(n + \frac{1}{2})$.

   (a) Give the state $\Psi(x, t)$ at a later time t.

   (b) For this state, determine the expectation value of the kinetic energy $E_k = \frac{p_{op}^2}{2m}$ as a function of time.

   (3p)

TURN PAGE!

3. **Two dimensional Square well**

A particle is placed in the potential (a 2 dimensional square well)

$$V(x,y) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

(a) Calculate (solve the Schrödinger equation) the eigenfunctions !

(b) Write down the eigenfunctions for the ground state and one for the lowest excited states. Formulate the meaning of orthogonallity and show by explicit calculation that these two eigenfunctions are orthogonal.

(3p)

4. **Hydrogen atom**

Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

$$\Psi(\mathbf{r}, t=0) = \frac{1}{\sqrt{15}} \left(3\psi_{100}(\mathbf{r}) - 2\psi_{210}(\mathbf{r}) + \psi_{310}(\mathbf{r}) - \psi_{322}(\mathbf{r})\right)$$

(a) Is this wave function an eigenfunction of the parity operator $\hat{\Pi}$?

(b) What is the probability of finding the system in the ground state? In the state (210)? In the state (310)? In the state (322)? In any other state?

(c) What is the expectation value of the energy (in eV); of the operator $\mathbf{L}^2$ (in units of $\hbar^2$); of the the operator L_z (in units of $\hbar$).

(3p)

5. **Spin**

Evaluate for a spin 1/2 particle described by the spinor χ the expectation values of the 3 cartesian components ($< S_x >, < S_y >, < S_z >$) of the spin and also their squares ($< S_x^2 >, < S_y^2 >, < S_z^2 >$)

$$\chi = \frac{1}{3} \begin{pmatrix} 2-i \\ 2 \end{pmatrix}.$$

(3p)

GOOD LUCK !