LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0047T**
Examination date	2018-09-01
Time	9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber   Tel: (49)2088, Room E163
Examiner: Hans Weber   Tel: (49)2088, Room E163

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 10.0, 5: 12.0

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1. **Misc.**

   a) Evaluate the commutator $[y^2, p_y^2]$.

   b) The ion Be^{3+} has the nuclear charge +4 but only one electron. How much energy does it take to excite the electron from the ground state to the level 2s? Give a numerical value in electron Volts (eV)!

   c) The wave function of a hydrogen atom in an eigenstate to the Hamilton operator is:

   $$\Psi(r,\theta,\phi) = \frac{1}{81\sqrt{6\pi}}(1/a_\mu)^{3/2}(r^2/a_\mu^2)e^{-r/3a_\mu}[3\cos^2\theta - 1],$$

   where a_μ is the Bohr radius (with the reduced mass). Determine the quantum numbers n, l och m_l.

(3p)

2. **Hydrogen atom**

   Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

   $$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{15}}\left(3\psi_{100}(\mathbf{r}) - 2\psi_{210}(\mathbf{r}) + \psi_{310}(\mathbf{r}) - \psi_{322}(\mathbf{r})\right)$$

   (a) Is this wave function an eigenfunction of the parity operator $\hat{\Pi}$?

   (b) What is the probability of finding the system in the ground state? In the state (210)? In the state (310)? In the state (322)? In any other state?

   (c) What is the expectation value of the energy (in eV); of the operator $\mathbf{L}^2$ (in units of $\hbar^2$); of the the operator L_z (in units of $\hbar$).

(3p)

TURN PAGE!

3. **Quantum rotator**

The Hamiltonian (in units of eV) for a given axially symmetric quantum rotator is (Pay attention to the subscripts)

$$H = \frac{L_x^2 + L_y^2}{2\hbar^2} + \frac{L_z^2}{3\hbar^2}$$

What are the possible energies? (3p)

4. **A quantum system at temperature**

A quantum system has **four** eigenstates with energies according to

$$E_{n_1,n_2} = (n_1 + n_2 + 1)\ \hbar\omega$$

where n_1, n_2 are integers $n_i = 0, 1$. The quantum system is coupled to a heatbath of temperature T with which it can exchange energy.

(a) Calculate the partition function of the system for any temperature.

(b) At what temperature T equals the probability to find the quantum system in a state of energy $\hbar\omega$ to find it in a state of energy $2\hbar\omega$?

(c) How large is this probability ?

(3p)

5. **Reflection and transmission at a potential step**

Consider an electron of energy E incident on the potential step $V(x)$,

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

where $V_0 = 4.5$ eV. Calculate the reflection coefficient R and the transmission coefficient T

a) when $E = 2.0$ eV,

b) when $E = 5.0$ eV,

c) when $E = 7.0$ eV.

(3p)

GOOD LUCK !