LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0047T**
Examination date	2019-03-21
Time	9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysikalia/Fysika, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 10.0, 5: 12.0

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1. **Harmonic oscillator**

   A particle is in a harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2 x^2$ and the energy is measured. The probability that the energy measurement yields $\frac{3}{2}\hbar\omega$ is 36% and the probability that the energy measurement yields $\frac{5}{2}\hbar\omega$ is 64%. The expectation value of the position $< x >$ is a minimum at time $t = 0$.

   (a) Find the time–dependent wave function.

   (b) Calculate the expectation value $< p >$ of the momentum for this particle, as a function of time.

   (c) Calculate the expectation value $< E >$ of the energy.

   (3p)

2. **Time evolution of a solution**

   A particle, of mass m, in a infinite square well potential has an initial wave function $\psi(x, t = 0) = Ax(a - x)$. The width of the well is a and A is a constant.

   a) Find the time evolution of the wave function $\psi(x, t)$

   b) Find the expectation value of the position as a function of time.

   (3p)

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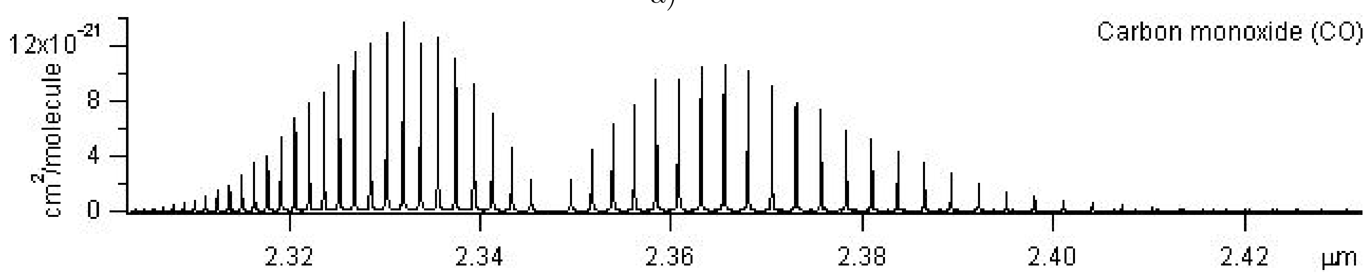
3. **Molecular spectra**

Carbon monoxide is the second-most common molecule in the interstellar medium, after molecular hydrogen. Interstellar CO was first detected with radio telescopes in 1970.

In the VPL Molecular Spectroscopic Database the two graphs for the Carbon monoxide molecular spectra shown in figure 1 are found. The spectra consist of both rotational and vibrational levels.

In the lectures only the vibrational transitions with $\Delta n = \pm 1, 0$ have been covered. In a real material however the potential is not purely harmonic (pure harmonic potential is $V(x) = kx^2$) but other anharmonic (like x^4) contributions exist. Due to these anharmonic contributions also vibrational transitions with higher changes in n are allowed ie $\Delta n = \pm 2$. These transitions are however much rarer as their transition probability is much lower.

a)



b)

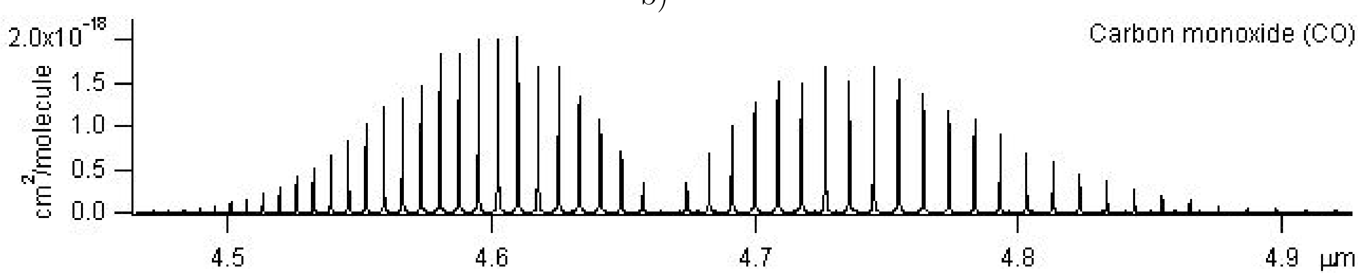


Figure 1: Two spectra for Carbon monoxide (CO) in the micro wave range. Shown is intensity as a function of wavelength. **Note:** the scale on the x–axis is in μm.

(a) Of the two spectra which one (upper/lower) belongs (determine by inspection) to the anharmonic vibrational transition and which one belongs to the harmonic vibrational transition? Motivate !

(b) With data taken from the (the harmonic vibrational spectra) graph calculate the distance between the two atoms of the molecule.

(c) Explain the apparent missing line in the middle of the spectra (the harmonic vibrational spectra).

(3p)

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4. **Quantum rotator**

The Hamiltonian (in units of eV) for a given axially symmetric quantum rotator is (Pay attention to the subscripts)

$$H = \frac{L_z^2 + L_y^2}{2\hbar^2} + \frac{L_x^2}{4\hbar^2}.$$

Write down an expression for the eigenenergys. How are the quantum numbers related ? (3p)

5. **A quantum system at temperature**

A quantum system has **four** eigenstates with energies according to

$$E_{n_1,n_2} = (n_1 + n_2 + 1)\,\hbar\omega$$

where n_1, n_2 are integers $n_i = 0, 1$. The quantum system is coupled to a heatbath of temperature T with which it can exchange energy.

(a) Calculate the partition function of the system for any temperature.

(b) At what temperature T equals the probability to find the quantum system in a state of energy $\hbar\omega$ to find it in a state of energy $2\hbar\omega$?

(c) How large is this probability ?

(3p)

GOOD LUCK !