LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	**F0047T**
Examination date	2019-08-31
Time	9.00 - 14.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysikalia/Fysika, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow.
Total number of problems: **5**. Maximum number of point is 15 p. 7.5 points are required to pass the examination. Grades 3: 7.5, 4: 10.0, 5: 12.0

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1. **Hydrogen atom**

   Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

$$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{15}} \left(3\psi_{100}(\mathbf{r}) - 2\psi_{200}(\mathbf{r}) + \psi_{320}(\mathbf{r}) - \psi_{322}(\mathbf{r})\right)$$

   (a) Is this wave function an eigenfunction of the parity operator $\hat{\Pi}$?

   (b) What is the probability of finding the system in the ground state? In the state (200)? In the state (320)? In the state (322)? In any other state?

   (c) What is the expectation value of the energy (in eV); of the operator $\mathbf{L}^2$ (in units of $\hbar^2$); of the the operator L_z (in units of $\hbar$).

(3p)

2. **Particle in one dimensional potential**

   A particle of mass m moves in one dimension under the influence of a potential $V(x)$. Suppose the particle is in an energy eigenstate $\psi(x) = \left(\frac{\gamma^2}{\pi}\right)^{\frac{1}{4}} e^{-\gamma^2 x^2/2}$ with energy $E = \frac{\hbar^2\gamma^2}{2m}$.

   (a) Calculate the expectation value of the position x, ($\langle x \rangle$).

   (b) Calculate the expectation value of the momentum p_x, ($\langle p_x \rangle$).

   (c) Find $V(x)$.

(3p)

TURN PAGE!

3. **Misc.**

a) Evaluate the commutator $[x^2, p_x^2]$.

b) Li^{2+} has the nuclear charge +3 but only one electron. How much energy does it take to excite the electron from the ground state to the level 2p? Give a numerical value in electron Volts (eV)!

c) The wave function of a hydrogen atom in an eigenstate to the Hamilton operator is:

$$\Psi(r,\theta,\phi) = \frac{2\sqrt{5}}{9\sqrt{6\pi}}(1/a_\mu)^{3/2}\frac{r}{a_\mu}\left(1-\frac{r}{6a_\mu}\right)e^{-r/3a_\mu}[3\cos^2\theta - 1],$$

where a_μ is the Bohr radius (with the reduced mass). Determine the quantum numbers n, l och m_l.

(3p)

4. **Operators and eigenfunctions**

Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

Note: Do not make guesses, a wrong answer will be counted negative ! (No answer to one of the items is counted as zero)

(a) $\psi(t) = \sin(\omega t)\cos(\omega t)$ and $\hat{A} = i\hbar\frac{\partial^2}{\partial t^2}$.

(b) $\psi(t) = \cos^2(\omega t) - \sin^2(\omega t)$ and $\hat{A} = i\hbar\frac{\partial^2}{\partial t^2}$.

(c) $\psi(x) = \sin kx$ and $\hat{A} = \frac{\partial}{\partial x}$.

(d) $\psi(x) = kx^2$ and $\hat{A} = \frac{\partial}{\partial x}$.

(e) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

(f) $\psi(x) = e^{ikx} + e^{-ikx}$ and $\hat{A} = \frac{\partial}{\partial x}$.

(g) $\psi(x) = \cos kx$ and $\hat{A} = \hat{P}$ = the parity operator.

(h) $\psi(z) = Ce^{-\omega z}$ and $\hat{A} = -i\frac{\hbar}{2}\frac{\partial}{\partial z}$.

(i) $\psi(z) = C(1 + z^3)$ and $\hat{A} = -i\hbar\frac{\partial}{\partial z}$.

(0 – 3 p)

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5. **Two dimensional Square well and parity**

A particle is placed in the potential (a 2 dimensional square well)

$$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

(a) Calculate (solve the Schrödinger equation !) the eigenfunctions !

(b) The Hamiltonian commutes with the parity operator P, $P\Psi(x,y) = \Psi(-x,-y) = \lambda\Psi(x,y)$ where the eigenvalue λ can take two possible values ± 1.

Write down the eigenstates corresponding to the four lowest **energies** in such a way that they are also eigenfunctions of the parity operator P. What is the parity of these states?

(3p)

GOOD LUCK !