LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	**F0047T**
Examination date	2020-01-14
Time	15.00 - 20.00 (5 hours)

Examination in: KVANTFYSIK / QUANTUM PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysika, Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of points is 15 p. 8.5 points are required to pass the examination. Grades 3: 8.5, 4: 10.5, 5: 12.0 . This includes the bonus points from the three home assignements.

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1. **Harmonic oscillator**

   Consider a three-dimensional harmonic oscillator, for which the potential is

   $$V(r) = \frac{1}{2} m\omega^2 r^2$$

   (a) Show that separation of variables in cartesian coordinates turns this into three one-dimensional oscillators, and exploit you knowledge of the latter to determine the allowed energies E. Use suitable quantum numbers to express the energies.

   (b) Write explicitly down the wave function of the ground state.

   (c) Determine the degeneracy $d(n)$ of the energies E.

   (3p)

2. **Two dimensional Square well and parity**

   A particle is placed in the potential (a 2 dimensional square well)

   $$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation !) the eigenfunctions !

   (b) The Hamiltonian commutes with the parity operator P, $P\Psi(x,y) = \Psi(-x,-y) = \lambda\Psi(x,y)$ where the eigenvalue λ can take two possible values ± 1.

   Write down the eigenstates corresponding to the four lowest **energies** in such a way that they are also eigenfunctions of the parity operator P. What is the parity of these states?

   (3p)

TURN PAGE!

3. **Spin**

   Suppose a spin 1/2 particle is in the state

$$\chi = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ 1+i \end{pmatrix}.$$

   (a) What are the probabilities of getting $+\hbar/2$ and $-\hbar/2$, if you mesure $S_{\mathbf{z}}$?

   (b) What are the probabilities of getting $+\hbar/2$ and $-\hbar/2$, if you mesure $S_{\mathbf{y}}$?

(3p)

4. **Parity operator**

   The parity operator $\hat{\Pi}$, acts on a function $\Psi(x)$ in the following way $\hat{\Pi}\Psi(x) = \Psi(-x) = \lambda\Psi(x)$ where the equality is valid only if the function Ψ is an eigenfunction to the operator $\hat{\Pi}$, the eigenvalue λ can take two possible values ± 1.

   (a) Show that the following functions are eigenfunctions of the parity operator and find the corresponding eigenvalue in each case.

      i. $\Psi_1(x) = C\left(\cos(\frac{\pi x}{L}) + \cos(\frac{3\pi x}{L})\right)$

      ii. $\Psi_2(x, y, z) = Ce^{-a\sqrt{x^2+3y^2+z^2}}$

      iii. $\Psi_3(r, \theta, \phi) = Cf(r)\left(\cos(\theta) + cos^3(\theta)\right)\ e^{i\phi}$

   (b) If $\psi_+(x, y, z)$ and $\psi_-(x, y, z)$ are eigenfunctions of $\hat{\Pi}$ corresponding to eigenvalues +1 and -1 respectively.

      i. Is the function $\Psi = 4\psi_+(x, y, z) + 2\psi_-(x, y, z)$ an eigenfunction of the parity operator?

      ii. Show that it is an eigenfunction of $\hat{\Pi}^2$, and find the eigenvalue.

      iii. Do the functions $e^{-\alpha x}$ and $e^{\alpha x}$ have parity? If yes what are their eigenvalues. If no form possible linear combinations that have a definite parity and what are their eigenvalues.

(3 p)

5. **Hydrogen atom**

   Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunctions $\psi_{nlm_l}(\mathbf{r})$:

$$\Psi(\mathbf{r}, t = 0) = A\left(2\psi_{100}(\mathbf{r}) - 3\psi_{211}(\mathbf{r}) + \psi_{320}(\mathbf{r}) - \psi_{322}(\mathbf{r})\right)$$

   (a) Determine A so that the wave function is normalised.

   (b) What is the probability of finding the system in the ground state? In the state (211)? In the state (320)? In the state (322)? In any other state?

   (c) What is the expectation value of the energy (in eV); of the operator $\mathbf{L}^2$ (in units of $\hbar^2$); of the the operator L_z (in units of $\hbar$).

(3p)

GOOD LUCK !