LULEÅ UNIVERSITY OF TECHNOLOGY

DIVISION OF PHYSICS
Niklas Lehto

2000-11-14

# The postulates of Quantum Mechanics

(a) The wave function or state function contains all information of a physical system.

(b) *The superposition principle*: It is possible to make linear superpositions of the dynamical states of a quantum system

(c) There is a linear and Hermitian operator $\hat{A}$ associated to every dynamical variable $\mathcal{A}$ (observable $\mathcal{A}$).

(d) The only result of a precise measurement of the dynamical variable $\mathcal{A}$ is one of the eigenvalues $a_n$ of the Hermitian operator $\hat{A}$ associated with $\mathcal{A}$. A measurement will always give a real value, since $\hat{A}$ is Hermitian

(e) If a series of measurements is made of the observable $\mathcal{A}$ on an ensamble of systems, described by the wave function $\psi$, the expectation value of this observable is

$$\left\langle \hat{A} \right\rangle = \frac{\left\langle \psi | \hat{A} | \psi \right\rangle}{\langle \psi | \psi \rangle}$$

(f) A wave function representing any dynamical state can be expressed as a linear combination of eigenfunctions of $\hat{A}$, where $\hat{A}$ is the operator associated with an observable.

From (e) and (f) it follows that if a system is described by the normalized state $|\psi\rangle$, then the probability that the eigenvalue $a_n$ will be found in a measurement of the observable $\mathcal{A}$ is given by

$$P(a_n) = |\langle u_n | \psi \rangle|^2 \,,$$

where $|u_n\rangle$ is an eigenstate of $\hat{A}$.

(g) The time evolution of the wave function of a system is determined by the time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi(t) = \hat{H} \psi(t),$$

where $H$ is the Hamiltonian, or the total energy operator of the system.

(h) If a measurement of the observable $\mathcal{A}$ on a system in a state $|\psi\rangle$ yields the eigenvalue $a_n$, then the system is projected by the measurement into the state $|u_n\rangle$, *i.e.* the new state is given by

$$|\psi'\rangle = \frac{|u_n\rangle \langle u_n | \psi \rangle}{\sqrt{< \psi | u_n > < u_n | \psi >}}.$$

The wave function collapse. A subsequent measurement of the observable $\mathcal{A}$ will give the value $a_n$.