LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	MTF131
Examination date Time	2005-12-17 09.00 - 14.00

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: 49 20 88, Room E111
Examiner: Hans Weber Tel: 49 20 88, Room E111
The results are put up: 9 January 2006 on the notice-board, building E
The marking may be scrutinised: after the results have been put up

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Particle in a one–dimensional box**

   A particle is confined to a one–dimensional infinite square well described by the potential $V(x)$,

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 \le x \le L \\ +\infty & \text{for} \quad x > L, \text{ or } x < 0 \end{cases}$$

   a) Calculate the normalised eigenfunctions $\psi_n(x)$ of the system.
   b) Calculate the eigenenergies E_n of the system.
   c) The size of the box is increased (to L') according to $L' = 3L$. If ν is the emited photon frequency for a transition from the lowest excited state to the ground state. What would the corresponding frequency ν' be for photon emision for a box of size L' (express your answer in terms of ν).

   (3p)

2. **van der Waals gas**

   The partition function Z for a gas of N interacting particles is given by

$$Z = \left(\frac{V - bN}{N}\right)^N \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{\frac{3N}{2}} e^{\frac{aN^2}{Vk_BT}}$$

   where a and b are constants and V is the volume. Derive the **equation of state** of the gas and also evaluate it's **energy** U.

   (3p)

3. **Binding of O_2 to hemoglobin**

A hemoglobin molecule can bind four O_2 molecules. Assume ϵ is the energy of each bound O_2, relative to O_2 at rest at infinite distance. Let λ denote the absolute activity $e^{\mu/\tau}$ of free O_2 (in solution).

(a) What is the probability that one and only one O_2 is adsorbed on a hemoglobin molecule?

(b) What is the probability that four O_2 are adsorbed on a hemoglobin molecule?

(c) Make a sketch of these probabilities as a function of λ.

(3p)

4. **Spin**

Evaluate for a spin 1/2 particle described by the spinor χ the expectation values of the 3 cartesian components ($< S_x >, < S_y >, < S_z >$) of the spin and also their squares ($< S_x^2 >, < S_y^2 >, < S_z^2 >$)

$$\chi = \frac{1}{3} \begin{pmatrix} 2-i \\ 2 \end{pmatrix}.$$

(3p)

5. **Quantum rotator**

The Hamiltonian (in units of eV) for a given axially symmetric quantum rotator is

$$H = \frac{L_x^2 + L_y^2}{2\hbar^2} + \frac{L_z^2}{3\hbar^2}$$

What are the possible energies? (3p)

GOOD LUCK !