LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

| Course code | MTF131 |
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| Examination date | 2006-10-24 |
| Time | 09.00 - 14.00 |

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber — Tel: 492088 or 0708-592088, Room E111
Examiner: Hans Weber — Tel: 492088 or 0708-592088, Room E111
The results are put up: 14 November 2006 on the notice-board, building E
The marking may be scrutinised: after the results have been put up

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator,COLLECTION OF FORMULAE

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Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Identical particles**

   Five identical particles are placed in the potential

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 \leq x \leq a \\ \\ +\infty & \text{for} \quad x > a \text{ and } x < 0. \end{cases}$$

   There is no interaction between the particles other than the fact that they are identical.

   (a) Calculate the energy of the ground state and the first excited state if the particles are identical spin 0 bosons.

   (b) Calculate the energy of the ground state and the first excited state if the particles are identical spin $\frac{1}{2}$ fermions.

(3 p)

2. **Spin**

   Evaluate, for a spin 1/2 particle described by the spinor $\chi$ the expectation values of the three cartesian components $(< S_x >, < S_y >, < S_z >)$ of the spin and also their squares $(< S_x^2 >, < S_y^2 >, < S_z^2 >)$

$$\chi = \frac{1}{3} \begin{pmatrix} 2 - 2i \\ 1 \end{pmatrix}.$$

(3p)

3. **Misc.**

   a) Evaluate the commutator $[x^2, p_x^2]$ .

b) $Li^{2+}$ has the nuclear charge +3 but only one electron. How much energy does it take to excite the electron from the ground state to the level 2s? Give a numerical value in electron Volts (eV)!

c) The wave function of a hydrogen atom in an eigenstate to the Hamilton operator is:

$$\Psi(r,\theta,\phi) = \frac{1}{81\sqrt{6\pi}}(1/a_\mu)^{3/2}(r^2/a_\mu^2)e^{-r/3a_\mu}[3\cos^2\theta - 1],$$

where $a_\mu$ is the Bohr radius (with the reduced mass). Determine the quantum numbers $n, l$ och $m_l$.

(3p)

4. **Hydrogen like spectra**

The Institutet för rymdfysik (IRF) in Kiruna has at the moment active instruments at four different planets in our solar system. One of the instruments detects the following spectra in ultra violett light emitted from a carbon rich area.

| $\lambda$ (nm) | 207.80 | 129.63 | 104.20 | 91.84 |
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At IRF they note that the lines listed above very much appear to be like a hydrogen spectra. It is suggested the spectra originates from highly ionized carbon with only one electron left and that the lines belong to the same series ie they all have the same lower level with principal quantum number $n$ and one may assume the upper levels are adjecent. Determine the principal quantum numbers for the levels involved in the transitions listed above.

(3p)

5. **Harmonic oscillator in two dimensions**

The energy levels of a two dimensional harmonic oscillator are characterized by 2 indices $n_x = 0, 1, 2, 3...\infty$ and $n_y = 0, 1, 2, 3...\infty$. The energy is given by $\epsilon_{n_x,n_y} = (n_x + n_y + 1)\hbar\omega$ The oscillator is coupled to a heat reservoir of temperature $\tau$.

**a)** Evaluate an expression for the partition function $Z_{osc}(\tau)$ and the Helmholtz free energy $F_{osc}(\tau)$.

**b)** Evaluate the specific heat $C_v$ of the oscillator. What is the low temperature and high temperature limit of $C_v$ draw a figure. Hint use the entropy $\sigma$ to evaluate $C_v$.

(3p)

GOOD LUCK !