LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	MTF131
Examination date Time	2006-12-19 09.00 - 14.00

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: 492088 or 0708-592088, Room E111
Examiner: Hans Weber Tel: 492088 or 0708-592088, Room E111
The results are put up: 14 January 2007 on the notice-board, building E
The marking may be scrutinised: after the results have been put up

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Operators and eigenfunctions**

   Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

   (a) $\psi(t) = \sin \omega t$ and $\hat{A} = i\hbar \frac{\partial}{\partial t}$.

   (b) $\psi(z) = C(1 + z^2)$ and $\hat{A} = -i\hbar \frac{\partial}{\partial z}$.

   (c) $\psi(z) = C_1 e^{ikz} + C_2 e^{-ikz}$ and $\hat{A} = -i\hbar \frac{\partial}{\partial z}$.

   (d) $\psi(z) = Ce^{-3z}$ and $\hat{A} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2}$.

   (e) $\psi(z) = Ce^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (f) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (3 p)

2. **Eigenfunctions and uncertainty**

   An electron confined in a quantum well has four discrete energy levels $E_1 = 0.27$ eV, $E_2 = 1.08$ eV, $E_3 = 3.65$ eV, $E_4 = 4.06$ eV. It is in a state in which the probabilities associated with these energies are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{16}$ and $\frac{1}{16}$ respectively.

   (a) Find the expectation value of its energy $\langle \hat{H} \rangle$ and the corresponding uncertainty $\Delta \hat{H}$.

   (b) Obtain an expression for the wave function $\Psi(z)$ describing the state of the particle in terms of its energy eigenfunctions $\psi_n(z)$ at time $t = 0$. Why is the expression not unique? Write down two different wave functions corresponding to the same values of $\langle \hat{H} \rangle$ and $\Delta \hat{H}$ that you found in (a).

   (3 p)

3. **Helium ^3He**

   Helium ^3He has spin $= \frac{1}{2}$ and may at low temperatures to a good approximation be described as an ideal Fermi gas. At these low temperatures ^3He is in the liquid phase with a density of $\rho = 83$ kg m^{-3}. Determine the Fermi temperature T_F and also the specific heat C_v of ^3He at T=0.5 K.

   (3 p)

4. **Two spinnless particles in an infinite well**

   Two identical non–interacting spinnless particles of mass m are trapped in the same infinetely deep one–dimensional square well of width L.

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions and write down the 5 lowest one particle energies !

   (b) Now take both particles into consideration. What are the two particle eigenfunctions and the five lowest enerigies of the two particle system?

   (c) Write down an expression for the partition function $Z(\tau)$.

   (3 p)

5. **Quantum mechanical rotor**

   A quantum mechanical rotor (molecule) has energy levels $\epsilon_j = j(j+1)\hbar^2/2I$ where I is the moment of inertia, each level has degeneration $g(j) = 2j+1$ where $j = 0, 1, 2, \ldots$. Calculate the for the rotational degrees of freedom the contribution to the heat capacity for low temperatures ($\tau << \hbar^2/I$).

   (3 p)

GOOD LUCK !