LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	MTF131
Examination date	2007-09-01
Time	09.00 - 14.00

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5

Teacher on duty: Hans Weber	Tel: 492088 or 0708-592088, Room E111
Examiner: Hans Weber	Tel: 492088 or 0708-592088, Room E111
The results are put up:	24 September 2007 on the notice-board, building E
The marking may be scrutinised:	after the results have been put up

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator,COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Two dimensional Rectangular well**

   A particle is placed in the potential (a 2 dimensional rectangular well)

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 \le x \le a \text{ and } 0 \le y \le \sqrt{2}a \\ +\infty & \text{for} \quad x > a,\ x < 0 \text{ and } y > \sqrt{2}a,\ y < 0. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions !

   (b) What are the 5 lowest eigenenergies ?

   (c) What are the degeneracies of these 5 lowest eigenstates ?

   (3p)

2. **van der Waals gas**

   The partition function Z for a gas of N interacting particles is given by

$$Z = \left(\frac{V - bN}{N}\right)^N \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{\frac{3N}{2}} e^{\frac{aN^2}{Vk_BT}}$$

   where a and b are constants and V is the volume. Derive the **equation of state** of the gas and also evaluate it's **energy** U.

   (3p)

3. **Helium** ^3He

   Helium ^3He has spin $= \frac{1}{2}$ and may at low temperatures to a good approximation be described as an ideal Fermi gas. At these low temperatures ^3He is in the liquid phase with a density of $\rho = 83$ kg m^{-3}. Determine the Fermi temperature T_F and also the specific heat C_v of ^3He at T=0.5 K.

   (3p)

4. **Diatomic molecule**

An ideal gas consists of N identical molecules. Each molecule consists of two atoms with the following rotational energy levels: $E(j) = j(j+1)\frac{\hbar^2}{2I}, \quad j = 0, 1, 2, ...$ Where I is the moment of inertia. Each level is $(2j+1)$ times degenerate. Determine to lowest order in temperature the contribution to C_v from the rotational degrees of freedom.

(3p)

5. **Angular momentum**

A particle is placed in a spherically symmetric potential $V(r)$. The particle is in a stationary state described by

$$\psi(\boldsymbol{r}) = \psi(x, y, z) = N(xy + zy)e^{-\alpha r},$$

where N and α are constants.

(a) A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities.

(b) Calculate the expectation values $\langle L^2 \rangle$ and $\langle L_z \rangle$.

(3p)

GOOD LUCK !